Summary of Baseline Real Business Cycle Model

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Abstract

This document provides a summary for the baseline real business cycle model presented in the file "Baseline_RBC_Model.pdf"

1 Model

Two sector economy resource constraint:

$$Y_t = C_t + I_t \tag{1}$$

Time resource constraint:

$$Z_t = 1 - L_t \tag{2}$$

Log Utility Function:

$$U\left(C_t, Z_t\right) = \log C_t + \sigma_L \log\left(Z_t\right) \tag{3}$$

Household budget constraint:

$$C_t + I_t = w_t L_t + r_t K_t \tag{4}$$

Capital accumulation equation:

$$K_{t+1} = \left(1 - \delta\right) K_t + I_t \tag{5}$$

Cobb-Douglas production function:

$$Y_t = \varepsilon_{A,t} K_t^{\alpha} L_t^{1-\alpha} \tag{6}$$

Law of motion for aggregate productivity shock:

$$\ln \varepsilon_{A,t} = \rho \ln \varepsilon_{A,t} + u_t,\; u_t \sim N\left(0, \sigma_A^2\right) \tag{7}$$

2 First Order Conditions

Household Euler consumption equation:

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left(\frac{1}{C_{t+1}} \left[r_{t+1} + (1-\delta) \right] \right) \tag{8}$$

Household labor supply equation:

$$Z_t = \frac{\sigma_L}{w_t} C_t \tag{9}$$

Resource constraints and capital accumulation equations:

$$Y_t = C_t + I_t \tag{10}$$
$$1 = Z_t + L_t \tag{11}$$
$$K_{t+1} = (1-\delta) K_t + I_t \tag{12}$$

Profit maximizing factor prices:

$$w_t = (1-\alpha) \frac{Y_t}{L_t} \tag{13}$$
$$r_t = \alpha \frac{Y_t}{K_t} \tag{14}$$

Production Function and law of motion for aggregate shock:

$$Y_t = \varepsilon_{A,t} K_t^{\alpha} L_t^{1-\alpha} \tag{15}$$
$$\ln \varepsilon_{A,t} = \rho \ln \varepsilon_{A,t} + u_t,\ u_t \sim N\left(0, \sigma_A^2\right) \tag{16}$$

3 Steady State Conditions

In order of FOCs:

$$\frac{1}{\beta} = \bar{r} + (1-\delta) \tag{17}$$
$$\bar{Z} = \frac{\sigma_L}{\bar{w}} \bar{C} \tag{18}$$
$$\bar{Y} = \bar{C} + \bar{I} \tag{19}$$
$$1 = \bar{Z} + \bar{L} \tag{20}$$
$$\delta \bar{K} = \bar{I} \tag{21}$$
$$\bar{w} = (1-\alpha) \frac{\bar{Y}}{\bar{L}} \tag{22}$$
$$\bar{r} = \alpha \frac{\bar{Y}}{\bar{K}} \tag{23}$$
$$\bar{Y} = \bar{\varepsilon}_A \bar{K}^{\alpha} \bar{L}^{1-\alpha} \tag{24}$$
$$\bar{\varepsilon}_A = \frac{1}{1-\rho} \tag{25}$$

Generally assume that $\bar{\varepsilon}_A = 1$ (i.e. $\rho = 0$ in steady state). Can collapse the above system into the following equations:

$$\frac{1-\bar{L}}{\bar{L}} = \sigma_L \frac{\beta^{-1} - 1 + (1-\alpha)\delta}{(1-\alpha)\left(\beta^{-1} - (1-\delta)\right)} \tag{26}$$

$$\bar{K} = \left[\frac{\alpha}{\beta^{-1} - 1 + \delta}\right]^{\frac{1}{1-\alpha}} \bar{L} \tag{27}$$

$$\bar{Y} = \bar{K}^{\alpha}\bar{L}^{1-\alpha} \tag{28}$$

$$\bar{C} = \bar{Y} - \delta\bar{K} \tag{29}$$

$$\bar{I} = \delta\bar{K} \tag{30}$$

$$\bar{Z} = 1 - \bar{L} \tag{31}$$

4 Calibration

Standard US calibrations on quarterly data:

$$\begin{aligned} \beta &= 0.99 \\ \sigma_L &= 1.75 \\ \delta &= 0.02 \\ \alpha &= \frac{1}{3} \\ \rho &= 0.95 \\ \sigma_A &= 1 \end{aligned}$$

4.1 Log-linearized system

In order of FOC's:

$$\begin{aligned} \hat{C}_t &= \mathbb{E}_t\left(\hat{C}_{t+1}\right) - \beta\bar{r}\mathbb{E}_t\left(1 + \hat{\varepsilon}_{A,t+1} + (\alpha-1)\hat{K}_{t+1} + (1-\alpha)\hat{L}_{t+1}\right) \\ \hat{Z}_t &= \hat{C}_t - \hat{\varepsilon}_{A,t} - \alpha\hat{K}_t + \alpha\hat{L}_t \\ \bar{Y}\hat{Y}_t &= \bar{C}\hat{C}_t + \bar{I}\hat{I}_t \\ 0 &= \bar{L}\hat{L}_t + \bar{Z}\hat{Z}_t \\ \hat{K}_{t+1} &= (1-\delta)\hat{K}_t + \hat{I}_t\delta \\ \hat{Y}_t &= \hat{\varepsilon}_{A,t} + \alpha\hat{K}_t + (1-\alpha)\hat{L}_t \\ \hat{\varepsilon}_{A,t} &= \rho\hat{\varepsilon}_{A,t-1} + u_t \end{aligned}$$

where we substituted out $\hat{w}_t$ and $\hat{r}_t$.

5 Estimating the model using Dynare

5.1 Model

See "RBC2.mod"

5.2 Linearized

See "RBC_LINEARIZED2.mod"

NOTE: Both mod files should produce the same steady state but they don't?!