Baseline Real Business Cycle Model

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Abstract

This document introduces the baseline real business cycle model at an applied level. By applied I mean that we will not be proving claimed properties of functions or the existence and uniqueness of equilibria. None of the infromation presented here is original. All sources of information are in the reference list.

1 Introduction

In this chapter we introduce the baseline real business cycle (RBC) model. The RBC model builds on the CKR model by adding two important features:

1. A household labor-leisure decision
   - Allows us to study the real life trade off between (real) wages and hours worked.
2. Uncertainty
   - Allows us to study the propogation of macroeconomic distirburbances within a general equilibrium framework.

A few points are worth noting:

1. The word “real" in real business cycle theory refers to the facts that the source of the fluctuations is real, rather than nominal, in nature. This assumption is in contrast to Keynesian analysis which views business cycles as being caused, at least in part, by nominal rigidities in the price or wage rates. One of the reasons for this is the debate between rational and adaptive expectations along with short run existence of market imperfections.
2. Although the term business cycle is historically interpreted as referring to the cyclical nature of in economic activity (i.e. peaks, troughs, expansions and recessions), RBC theory focuses more on the intertemporal co-movement between macroeconomic variables. The main emphasis of RBC theory is that consumption and investment are pro-cyclical, that real wages are pro-cyclical, and that goods prices are counter-cyclical.
3. Since it is an extension of the neoclassical growth model investment and capital accumulation still provide the key mechanisms through which newly added productivity shocks endogenously propagate through time. This propagation mechanism is what we refer to as the “business cycle”.

In what follows we introduce the microfoundations of the basic RBC model, we then look at how RBC models can be parametrized, solved and used in practice. To this end we present an approximate solution using perturbation methods along with the method of undetermined coefficients. The solution is then used to discuss its strengths and weaknesses. As we will see, the model's implications for labor market outcomes constitute a key weakness.

2 Microfoundations

We assume a two sector economy in which households and firms interact without any governments or international sector. Thus, the national income identity or resource constraint is given by:

$$Y_t = C_t + I_t, \tag{1}$$

where Y_t denotes output/income/expenditure, C_t denotes consumption and I_t denotes investment. We now define the household and firm behavior in this simple setting.

2.1 Households

To keep things simple the RBC literature typically assumes that the household sector of the economy is populated by identical and infinite living agents which are distributed over a continuum $i \in [0, 1]$. Since all agents are equivalent we can then reduce the problem by looking at the behaviour of a single agent from this continuum, called the "representative household". In each period the representative household chooses how much of their time they wish to allocate to labor: $L_{i,t}$, and leisure: $Z_{i,t}$, where the subscript i means that we are looking at a representative household $i \in [0, 1]$. If we normalize the total time in each period equal to one, then the households time constraint is given by:

$$Z_{i,t} = 1 - L_{i,t}. \tag{2}$$

In words, (2) says that the in a given period the agents total leisure time is given by the total time they have (in this case 1 unit) less the time they spend at work supplying labor. The standard RBC assumption is that households gain utility through leisure and receive disutility from working.

The problem for the representative household is to choose how much time to spend on leisure: $Z_{i,t}$, and how much to consume: $C_{i,t}$, in each time period t in order to maximize their lifetime utility. Mathematically, the representative household wants to solve the following problem:

$$\max_{\{C_{i,t}, Z_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_{i,t}, Z_{i,t}\right), \tag{3}$$

where $\beta \in (0, 1)$. In words, (3) says that the household chooses sequences of consumption and leisure to maximize the expected discounted value of their lifetime utility. The parameter β is the households subjective discount factor. A value of β close to zero means that the household puts a lot of weight on current consumption and is therefore impatient. Converesely, a value of β close to one implies that the household places more weight on future consumption and is therfore patient.

In a finite world, the level of consumption that a household can choose is constrained by their household income. Assumming that households own the capital stock, the households maximization problem is subject to their household budget constraint:

$$C_{i,t} + I_{i,t} = w_t L_{i,t} + r_t K_{i,t}, \tag{4}$$

where w_t is the real wage rate on supplying labor, r_t is the real rental rate of capital on supplying capital to firms and I_t is the level of household investment. The accounting identity in (4) simply says that houseolds expenditures (the LHS) must equate to household income (the RHS). In other words, there is no credit in this simple framework. It's also important to note that factor payments (i.e. w_t and r_t) are common to all hourseholds and expressed in real terms. Thus, firms will pay each household the same quantity of real goods for use of the capital and labor, implying that (1) there is no money in this simple framework and that (2) factors of production (i.e. K and L) opperate on perfectly competetive markets.

A key implication of equation (1) is that if households choose not to consume it some period t then they invest it. Mathematically we have:

$$I_{i,t} = Y_{i,t} - C_{i,t}, \tag{5}$$

In other words, the level of household invesment: I_t, in some period t is the equal to the quantity of household income: Y_t, that households chooses not to consume.

Since there is only one asset in this economy, namely capital, whenever households invest they raise more capital. Of course, capital in each period will errode or depreciate at some rate. This gives rise to the familiar capital accumulation equation from the Solow-Swan growth model:

$$K_{i,t+1} = (1-\delta) K_{i,t} + I_{i,t}, \tag{6}$$

where $\delta \in (0,1)$ denotes the depreciation rate.

Putting all of this together, given an initial stock of capital $K_0 > 0$, the households decision problem can be expressed as follows:

$$\begin{array}{ccc} & \max_{\{C_{i,t},L_{i,t},K_{i,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, 1-L_{i,t}) & \\ & \text{subject to} & \\ C_{i,t} + K_{i,t+1} & = & w_t L_{i,t} + r_t K_{i,t} + (1-\delta) K_{i,t} \end{array} \tag{7}$$

where the constraint in (7) comes from substituting the capital accumulation equation in (6) into the household budget constraint in (4). Also note that we have made use of the time resource constraint ($Z_t = 1 - L_t$) in the utility function. In words, (7) says that the representative household chooses sequences of consumption, labor and capital in order to maximize the discount (present) value of their lifetime utility subject to the economies resource constraint and capital accumulation equation.

To solve this problem we can use the method of Lagrange multiplers. The Lagrangian is given by:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_{i,t}, Z_{i,t}) + \lambda_t \left(w_t L_{i,t} + r_t K_{i,t} + (1-\delta) K_{i,t} - C_{i,t} - K_{i,t+1} \right) \right], \tag{8}$$

where λ_t is the lagrange multipler. The first order conditions (FOCs) are:

$$\frac{\partial \mathcal{L}}{\partial C_{i,t}} = \frac{\partial U(C_{i,t}, 1-L_{i,t})}{\partial C_{i,t}} - \lambda_t = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = \frac{\partial U(C_{i,t}, 1-L_{i,t})}{\partial L_{i,t}} - \lambda_t w_t = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = -\lambda_t + \mathbb{E}_t \beta \lambda_{t+1} \left[r_{t+1} - (1-\delta) \right] = 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = w_t L_{i,t} + r_t K_{i,t} + (1-\delta) K_{i,t} - C_{i,t} - K_{i,t+1} = 0 \tag{12}$$

Taking λ_t to the RHS in (9) gives:

$$\lambda_t = \frac{\partial U\left(C_{i,t}, 1-L_{i,t}\right)}{\partial C_{i,t}}. \tag{13}$$

Substituting (13) into (10) gives:

$$\frac{\partial U\left(C_{i,t}, 1-L_{i,t}\right)}{\partial C_{i,t}} w_t = \frac{\partial U\left(C_t, 1-L_t\right)}{\partial L_t}. \tag{14}$$

Also, substituting (13) into (11) gives:

$$\frac{\partial U\left(C_{i,t}, 1-L_{i,t}\right)}{\partial C_{i,t}} = \mathbb{E}_t \beta \frac{\partial U\left(C_{i,t+1}, 1-L_{i,t+1}\right)}{\partial C_{i,t+1}} \left[r_{t+1} + (1-\delta)\right]. \tag{15}$$

Finally, equation (12) recovers the resource constraint with embedded capital accumulation equation:

$$C_{i,t} + K_{i,t+1} = w_t L_{i,t} + r_t K_{i,t} + (1-\delta) K_{i,t}. \tag{16}$$

Equations (14) and (15) have the interpretation of being the households labor supply equation and Euler consumption equation respectively. Economically, the households labor supply equation expresses the labor supply decision as a function of the real wage rate and consumption. Next, the Euler consumption equation, captures the intertemporal tradeoff between consumption today and consumption tomorrow. Economically, it says that the household must be indifferent between consuming one more unit today and saving that unit and consuming it tomorrow.

2.1.1 Household Problem with Log Preferences

We can add a particular parametric structure to the households problem by assuming that households have a simple utility function:

$$U\left(C_{i,t}, 1-L_{i,t}\right) = \log C_{i,t} + \sigma_L \log\left(1-L_{i,t}\right), \tag{17}$$

where σ_L is a curvature parameter on labor.

Under this specific utility function, the partial derivatives arising from the households optimization problem are given by:

$$\frac{\partial U\left(C_{i,t}, 1-L_{i,t}\right)}{\partial C_{i,t}} = \frac{1}{C_{i,t}}, \tag{18}$$

$$\frac{\partial U\left(C_{i,t}, 1-L_{i,t}\right)}{\partial L_{i,t}} = \sigma_L \frac{1}{1-L_{i,t}}. \tag{19}$$

Thus, the households labor supply equation is given by:

$$\frac{1}{C_{i,t}} w_t = \sigma_L \frac{1}{1-L_{i,t}}. \tag{20}$$

Which can be written as:

$$Z_{i,t} = \frac{\sigma_L}{w_t} C_{i,t}. \tag{21}$$

In words, (21) says that the households leisure decision varies positively with the real wage rate and negatively to consumption. In other words, due to the decreasing marginal utility of consumption, the more a household consumes, the less labor they supply.

Next, the households Euler consumption equation is given by:

$$\frac{1}{C_{i,t}} = \mathbb{E}_t \beta \frac{1}{C_{i,t+1}} \left[r_{t+1} + (1-\delta) \right]. \tag{22}$$

Equivalently,

$$\frac{\mathbb{E}_t C_{i,t+1}}{C_{i,t}} = \mathbb{E}_t \beta \left[r_{t+1} + (1-\delta) \right]. \tag{23}$$

Equation (23) says that the growth rate of consumption (the LHS of (23)) is the expected return on discounted savings (the RHS of (23)). Notice that for smaller values of β, the more the household consumers today and consumption growth becomes smaller. On the other hand, as β increases, the household consumes less today and consumption growth becomes larger.

2.2 Firms

As was the case with the household sector, the RBC literature typically assumes that the firm sector of the economy is populated by identical and infinite living agents which are distributed over a continuum $j \in [0,1]$. Since all agents are equivalent we can then reduce the problem by looking at the behaviour of a single agent from this continuum, called the "representative firm". In each period the representative firm chooses how much capital: $K_{j,t}$, and labor: $L_{j,t}$ to employ from households to produce output: $Y_{j,t}$, using a Neoclassical production function:

$$Y_{j,t} = \varepsilon_{A,t} F\left(K_{j,t}, L_{j,t}\right), \tag{24}$$

where $\varepsilon_{A,t}$ is an aggregate productivity shock which eveolves according to the law of motion:

$$\ln \varepsilon_{A,t} = \rho \ln \varepsilon_{A,t} + u_t, \tag{25}$$

where $\rho \in (0,1)$ and $u_t \sim N\left(0, \sigma_A^2\right)$. The assumption of stochastic disturbances following an $AR(1)$ process is standard in the RBC literature. Note that the productivity shock has no subscript meaning that it impacts each firm in an equal manner.

The firms decision problem can be expressed in two equivalent ways:

1. They can maximize profits; or
2. They can minimize input costs.

Here we explore the first case and note that the second case yields the same results (you should verify this yourself). In the first case, firms chooses how much capital: $K_{j,t}$, and labor: $L_{j,t}$ to employ from households and output: $Y_{j,t}$ to produce, in order to maximize profits: $\Pi_{j,t}$, subject to their supply constraint in (24). Mathematically:

$$\begin{aligned} &\max_{Y_{j,t}, L_{j,t}, K_{j,t}} \Pi_{j,t} = \max_{L_{j,t}, K_{j,t}} \left\{ Y_{j,t} - w_t L_{j,t} - r K_{j,t} \right\} \\ &\text{subject to} \\ &Y_{j,t} = \varepsilon_{A,t} F\left(K_{j,t}, L_{j,t}\right) \end{aligned} \tag{26}$$

where $Y_{j,t}$ denotes the level of real output, and $w_t L_{j,t} + rK_{j,t}$ denotes the firms level of real costs on factor inputs. The Lagrangian for this (static) problem is:

$$\mathcal{L} = Y_{j,t} - w_t L_{j,t} - rK_{j,t} + \lambda_t \left[\varepsilon_{A,t} F\left(K_{j,t}, L_{j,t}\right) - Y_{j,t}\right], \tag{27}$$

where λ_t is the lagrange multipler on the supply constraint and can be interpreted as marginal profit.. The first order conditions (FOCs) are:

$$\frac{\partial \mathcal{L}}{\partial Y_{j,t}} = 1 - \lambda_t = 0 \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial L_{j,t}} = -w_t + \lambda_t \varepsilon_{A,t} \frac{\partial F\left(K_{j,t}, L_{j,t}\right)}{\partial L_{j,t}} = 0 \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial K_t} = -r_t + \lambda_t \varepsilon_{A,t} \frac{\partial F\left(K_{j,t}, L_{j,t}\right)}{\partial K_{j,t}} = 0 \tag{30}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = \varepsilon_{A,t} F\left(K_{j,t}, L_{j,t}\right) - Y_{j,t} = 0 \tag{31}$$

Solving (28) for λ_t gives:

$$\lambda_t = 1. \tag{32}$$

Substituting (32) into (29)and (30) and solving for the factor prices gives:

$$w_t = \varepsilon_{A,t} \frac{\partial F\left(K_{j,t}, L_{j,t}\right)}{\partial L_{j,t}}, \tag{33}$$

$$r_t = \varepsilon_{A,t} \frac{\partial F\left(K_{j,t}, L_{j,t}\right)}{\partial K_{j,t}}, \tag{34}$$

which says that maximium profit is attained when the factor prices are equal to their repsective marginal products.

2.2.1 Firm Problem with Cobb-Douglas Production

We can add a particular parametric structure to the firms problem by assuming that they have a simple Cobb-Douglas production function:

$$Y_{j,t} = \varepsilon_{A,t} K_{j,t}^{\alpha} L_{j,t}^{1-\alpha}. \tag{35}$$

where $\alpha \in (0,1)$ is a curvature parameter denoting capitals share of output.

Under this specific production function, the partial derivatives arising from the firms optimization problem are given by:

$$\frac{\partial F\left(K_{j,t}, L_{j,t}\right)}{\partial L_{j,t}} = (1-\alpha)\, \varepsilon_{A,t} K_{j,t}^{\alpha} L_{j,t}^{-\alpha} = (1-\alpha) \frac{Y_{j,t}}{L_{j,t}}, \tag{36}$$

$$\frac{\partial F\left(K_{j,t}, L_{j,t}\right)}{\partial K_{j,t}} = \alpha \varepsilon_{A,t} K_{j,t}^{\alpha-1} L_{j,t}^{1-\alpha} = \alpha \frac{Y_{j,t}}{K_{j,t}}. \tag{37}$$

Thus, the factor prices in (33) and (34) are given by:

$$w_t = (1-\alpha) \frac{Y_{j,t}}{L_{j,t}}, \tag{38}$$

$$r_t = \alpha \frac{Y_{j,t}}{K_{j,t}}. \tag{39}$$

3 Aggregation and Equilibrium Conditions

The economy is populated by two types of economic agents: j households and i firms, whose interactions produce market clearing conditions for goods, labour and physical capital markets.

In the first instance, the equilibrium condition for the goods market is given by:

$$\int_0^1 Y_{i,t} di = \int_0^1 C_{j,t} dj + \int_0^1 I_{j,t} dj, \tag{40}$$

which says that aggregate supply from firms equates to aggregate demand from households, where aggregate demand is given by the sum of aggregate consumption and aggregate investment. Letting Y_t, C_t and I_t denote the aggregate supply, consumption and investment respectively gives the resource constraint:

$$Y_t = C_t + I_t, \tag{41}$$

Next, the equilibrium condition for the labor and capital markets are given by:

$$L_t = \int_0^1 L_{i,t} di = \int_0^1 L_{j,t} dj \tag{42}$$

$$K_t = \int_0^1 K_{i,t} di = \int_0^1 K_{j,t} dj \tag{43}$$

Taken together, these equations imply that the aggregate production function for firms is given by:

$$Y_t = F(K_t, L_t), \tag{44}$$

and that the factor prices are given by:

$$w_t = \varepsilon_{A,t} \frac{\partial F(K_t, L_t)}{\partial L_t}, \tag{45}$$

$$r_t = \varepsilon_{A,t} \frac{\partial F(K_t, L_t)}{\partial K_t}. \tag{46}$$

4 Steady State

In this section we solve the model at a particular point, the non-stochastic steady state. The non-stochastic steady state describes a resting point in which all stochastic (i.e. random) elements have been removed. Thus, the non-stochastic steady state can be described as the limit of the stochastic steady state as the standard deviation of TFP, σ_A converges to zero. Once we have the non-stochastic steady-state we can use the relationships for calibration of the models parameters and then solve for equations that explain the models behaviour around this point.

To this end we assume that the household has a log utility function and that the firms have a Cobb-Douglas production function. The model conditions are given by:

$$\begin{aligned} 1 &= Z_t + L_t, \\ Y_t &= C_t + I_t, \\ Y_t &= w_t L_t + r_t K_t \end{aligned}$$

$$\begin{aligned}
K_{t+1} &= (1-\delta) K_t + I_t, \\
\frac{\sigma_L}{w_t} C_t &= Z_t, \\
\frac{\mathbb{E}_t C_{t+1}}{C_t} &= \mathbb{E}_t \beta \left[r_{t+1} + (1-\delta) \right], \\
Y_t &= \varepsilon_{A,t} K_t^{\alpha} L_t^{1-\alpha}, \\
\varepsilon_{A,t} &= \rho \varepsilon_{A,t-1} + u_t, \\
\frac{K_t}{L_t} &= \frac{\alpha}{(1-\alpha)} \frac{w_t}{r_t}, \\
w_t &= (1-\alpha) \frac{Y_t}{L_t}, \\
r_t &= \alpha \frac{Y_t}{K_t}.
\end{aligned}$$

The steady-state condistions are derived as follows:

1. Time resource constraint:

$$\begin{aligned}
L_t + Z_t &= 1 \\
\bar{L} + \bar{Z} &= 1
\end{aligned}$$

2. 2-sector economy resource constraint:

$$\begin{aligned}
Y_t &= C_t + I_t \\
\bar{Y} &= \bar{C} + \bar{I}
\end{aligned}$$

3. Budget Constraint:

$$\begin{aligned}
Y_t &= w_t L_t + r_t K_t \\
\bar{Y} &= \bar{w} \bar{L} + \bar{r} \bar{K}
\end{aligned}$$

4. Capital Accumulation Equation:

$$\begin{aligned}
K_{t+1} &= (1-\delta) K_t + I_t \\
\bar{K} &= (1-\delta) \bar{K} + \bar{I} \\
\bar{I} &= \delta \bar{K}
\end{aligned}$$

5. The labor supply rule

$$\begin{aligned}
Z_t &= \frac{\sigma_L}{w_t} C_t \\
\bar{Z} &= \frac{\sigma_L}{\bar{w}} \bar{C}
\end{aligned}$$

6. Production Function:

$$
\begin{aligned}
Y_t &= \varepsilon_{A,t} K_t^{\alpha} L_t^{1-\alpha} \\
\bar{Y} &= \bar{\varepsilon}_A \bar{K}^{\alpha} \bar{L}^{1-\alpha}
\end{aligned}
$$

7. The law of motion for productivity shock:

$$
\begin{aligned}
\ln(\varepsilon_{A,t}) &= \rho \ln(\varepsilon_{A,t}) + \varepsilon_{t+1} \\
\bar{\varepsilon}_A &= \rho \bar{\varepsilon}_A + 1 \\
\bar{\varepsilon}_A &= \frac{1}{1-\rho} \\
\bar{\varepsilon}_A &= 1, \text{ iff } \rho = 0
\end{aligned}
$$

We typically make this assumption in the steady state.

8. The Euler Consumption equation

$$
\begin{aligned}
\frac{\mathbb{E}_t C_{t+1}}{C_t} &= \mathbb{E}_t \beta \left[r_{t+1} + (1-\delta) \right] \\
1 &= \beta \left[\bar{r} + (1-\delta) \right] \\
\frac{1}{\beta} &= \bar{r} + (1-\delta)
\end{aligned}
$$

9. Wage rate:

$$
\begin{aligned}
w_t &= (1-\alpha) \frac{Y_t}{L_t} \\
\bar{w} &= (1-\alpha) \frac{\bar{Y}}{\bar{L}}
\end{aligned}
$$

10. Rental rate:

$$
\begin{aligned}
r_t &= \alpha \frac{Y_t}{K_t} \\
\bar{r} &= \alpha \frac{\bar{Y}}{\bar{K}}
\end{aligned}
$$

To summarise, in steady state we have the following conditions:

$$
\begin{aligned}
\bar{Y} &= \bar{C} + \bar{I}, && (47) \\
\bar{Y} &= \bar{w}\bar{L} + \bar{r}\bar{K}, && (48) \\
1 &= \bar{Z} + \bar{L}, && (49)
\end{aligned}
$$

$$
\begin{aligned}
\delta\bar{K} &= \bar{I}, && (50)\\
\bar{Z} &= \frac{\sigma_L}{\bar{w}}\bar{C}, && (51)\\
\frac{1}{\beta} &= \bar{r} + (1-\delta), && (52)\\
\bar{Y} &= \bar{K}^{\alpha}\bar{L}^{1-\alpha}, && (53)\\
\bar{w} &= (1-\alpha)\frac{\bar{Y}}{\bar{L}}, && (54)\\
\bar{r} &= \alpha\frac{\bar{Y}}{\bar{K}}. && (55)
\end{aligned}
$$

Importantly, if we have a unique value of $\bar{L}$ then we can derive unique values of $\bar{Y}$, $\bar{K}$, $\bar{C}$, $\bar{Z}$ and $\bar{I}$. Specifically, first note that if we write (52) in term of $\bar{r}$ and then substitute the result into (55) we get:

$$
\bar{Y} = \frac{\left(\beta^{-1} - (1-\delta)\right)}{\alpha}\bar{K}. \tag{56}
$$

Next, by solving (51) for $\bar{w}$ and equating the result with (54) then solving for $\bar{L}$ we get:

$$
\frac{1-\bar{L}}{\bar{L}} = \frac{\sigma_L}{(1-\alpha)}\frac{\bar{C}}{\bar{Y}}. \tag{57}
$$

Combining (57) with (48), (50) and (56) and doing some algebra gives:

$$
\frac{1-\bar{L}}{\bar{L}} = \sigma_L\frac{\beta^{-1} - 1 + (1-\alpha)\delta}{(1-\alpha)\left(\beta^{-1} - (1-\delta)\right)}. \tag{58}
$$

When this equation holds we will have a unique value of $\bar{L}$ from which we can derive unique values of $\bar{Y}$, $\bar{K}$, $\bar{C}$, $\bar{Z}$ and $\bar{I}$. Specifically, we have:

$$
\begin{aligned}
\bar{Y} &= \bar{K}^{\alpha}\bar{L}^{1-\alpha}\\
\bar{K} &= \left[\frac{\alpha}{\beta^{-1} - 1 + \delta}\right]^{\frac{1}{1-\alpha}}\bar{L}\\
\bar{C} &= \bar{Y} - \delta\bar{K}\\
\bar{I} &= \delta\bar{K}\\
\bar{Z} &= 1 - \bar{L}
\end{aligned}
$$

We will return to these equations when implementing the codes to estimate and simulate the model.

5 Calibration

Calibration refers to the process of the researcher choosing the models parameters. This choice should not be arbitrary but based on moment-matching from the sample data. The reason we don't estimate the parameter values using MLE for example is because of simultaneity in macroeconomic variables. Also, the fact that we have a single source of randomness our model is stochastically singular.

To calibrate α note that from (54) we have:

$$\alpha = 1 - \frac{\bar{L}\bar{w}}{\bar{Y}}$$

where $\frac{\bar{L}\bar{w}}{\bar{Y}}$ denotes labors share of income. Thus, if labors share of income is $\frac{2}{3}$ (generally true in the US) then $\alpha = \frac{1}{3}$.

Next, δ is given by:

$$\delta = \frac{\bar{I}}{\bar{K}}$$

which using annual US data suggests that $\delta \approx 7.6\%$ or using quarterly data about $(1-0.076)^{\frac{1}{4}} \approx \frac{1}{4}(0.076) \approx 2\%$ where we use a log transformation.

Next, since net return on investment is given by $\bar{r} = \alpha\frac{\bar{Y}}{\bar{K}} - \delta$, the quarterly discount factor is given by:

$$\beta = \left(\frac{1}{1+R}\right)^{\frac{1}{4}}$$

which, given a real return on bonds at about 4% suggests that $\beta \approx 0.99$.

Next, to solve for ρ we use the Solow-residual:

$$\varepsilon_{A,t} = \frac{Y_t}{K_t^{\alpha}L_t^{1-\alpha}},$$

along with the steady-state condition:

$$\bar{\varepsilon}_A = \frac{1}{1-\rho},$$

to get:

$$\rho = 1 - \frac{\bar{K}^{\alpha}\bar{L}^{1-\alpha}}{\bar{Y}},$$

which suggests that $\rho \approx 0.95$. To solve for σ_A we can set it equal to the standard deviation of detrended output from the data. Finally, to solve for σ_L we set it equal to the solution of (57).

5.1 Perturbation: An Approximate Solution Strategy

So far we have solved the model at a given point, the non-stochastic steady state. We are really interested in the solution for all possible values of capital and productivity. We now approximate the models saddle-arm or balanced growth path around the non-stochastic steady state. To this end we take a first-order log approximation of a each function at our non-stochastic steady-state. This method, known as pertubation, is common in the RBC liteature.

We begin by showing how to take a first-order log approximation of a function at a particular point and then apply the methods to our non-stochastic steady-state.

Consider the function:

$$f(x_t, y_t) = 0$$

where $f : \mathbb{R}^2 \to \mathbb{R}$ with continuous first and second derivatives and $x_t, y_t > 0$. We can restate this function without changing its key characteristics as:

$$f\left(e^{\ln(x_t)}, e^{\ln(y_t)}\right) = 0$$

We now take a first-order Taylor-Series approximation around the known steady-state $\ln(\bar{x})$and $\ln(\bar{y})$ of this general equation:

$$
\begin{aligned}
H(\bar{x},\bar{y}) + \frac{\partial f}{\partial \bar{x}}\bar{x}\left(\ln(x_t) - \ln(\bar{x})\right) + \frac{\partial f}{\partial \bar{y}}\bar{y}\left(\ln(y_t) - \ln(\bar{y})\right) &= 0 \\
\frac{\partial f}{\partial \bar{x}}\bar{x}\left(\ln(x_t) - \ln(\bar{x})\right) + \frac{\partial f}{\partial \bar{y}}\bar{y}\left(\ln(y_t) - \ln(\bar{y})\right) &= 0 \\
\frac{\partial f}{\partial \bar{x}}\bar{x}\hat{x}_t + \frac{\partial f}{\partial \bar{y}}\bar{y}\hat{y}_t &= 0
\end{aligned}
$$

where we use the fact that in steady state $H(\bar{x},\bar{y}) = 0$ and define $\hat{x}_t = \ln(x_t) - \ln(\bar{x}) = \ln\left(\frac{x_t}{\bar{x}}\right)$ and $\hat{y}_t = \ln(y_t) - \ln(\bar{y}) = \ln\left(\frac{y_t}{\bar{y}}\right)$. Note that $\hat{x}_t$ and $\hat{y}_t$ are interpreted as percentage deviations from the steady state.

As a practical manner note that following 'tricks' in solving linear in log type equations:

1. We can multiple and divide through by the steady state parameters and take advantage of known steady state relationships
2. We can use the approximation $x_t \approx \bar{x}(1 + \hat{x}_t)$
3. We can use the approximation $e^{x_t} \approx 1 + \hat{x}_t$

We are now ready to log-linearize our competitive equilibrium. First, categorize each of the equations into either linear in levels, linear in logs or non-linear type equations:

First, categorize each of the equations into either linear in levels, linear in logs or non-linear type equations:

1. Linear in levels

$$
\begin{aligned}
Y_t &= C_t + I_t \\
Y_t &= w_t L_t + r_t K_t \\
K_{t+1} &= (1-\delta) K_t + I_t \\
L_t + Z_t &= 1
\end{aligned}
$$

2. Linear in logs

$$
\begin{aligned}
Y_t &= \varepsilon_{A,t} K_t^{\alpha} L_t^{1-\alpha} \\
\ln \varepsilon_{A,t} &= \rho \ln \varepsilon_{A,t-1} + u_t \\
L_t &= 1 - \frac{\sigma_L}{w_t} C_t \\
w_t &= (1-\alpha)\varepsilon_{A,t} K_t^{\alpha} L_t^{-\alpha} \\
r_t &= \alpha \varepsilon_{A,t} K_t^{\alpha-1} L_t^{1-\alpha}
\end{aligned}
$$

3. Non-linear

$$
\frac{1}{C_t} = \mathbb{E}_t \beta \left[\frac{1}{C_{t+1}} \left(r_{t+1} - (1-\delta) \right) \right]
$$

Making use of the steady state conditions we can log-linearize the model as follows:

1. Linear in levels

   (a) 2-Sector economy resource constraint: $\bar{Y}\hat{Y}_t = \bar{C}\hat{C}_t + \bar{I}\hat{I}_t$

$$\begin{aligned} Y_t &= C_t + I_t \\ \bar{Y}\left(1+\hat{Y}_t\right) &= \bar{C}\left(1+\hat{C}_t\right) + \bar{I}\left(1+\hat{I}_t\right) \\ \bar{Y} + \bar{Y}\hat{Y}_t &= \bar{C} + \bar{I} + \bar{C}\hat{C}_t + \bar{I}\hat{I}_t \\ &= \bar{Y} + \bar{C}\hat{C}_t + \bar{I}\hat{I}_t \\ \bar{Y}\hat{Y}_t &= \bar{C}\hat{C}_t + \bar{I}\hat{I}_t \end{aligned}$$

   (b) Budget Constraint: $\bar{Y}\hat{Y}_t = \bar{w}\bar{L}\hat{w}_t\hat{L}_t + \bar{r}\bar{K}\hat{r}_t\hat{K}_t$

$$\begin{aligned} Y_t &= w_t L_t + r_t K_t \\ \bar{Y}\left(1+\hat{Y}_t\right) &= \bar{w}\bar{L}\left(1+\hat{w}_t\hat{L}_t\right) + \bar{r}\bar{K}\left(1+\hat{r}_t\hat{K}_t\right) \\ \bar{Y} + \bar{Y}\hat{Y}_t &= \bar{w}\bar{L} + \bar{r}\bar{K} + \bar{w}\bar{L}\hat{w}_t\hat{L}_t + \bar{r}\bar{K}\hat{r}_t\hat{K}_t \\ &= \bar{Y} + \bar{w}\bar{L}\hat{w}_t\hat{L}_t + \bar{r}\bar{K}\hat{r}_t\hat{K}_t \\ \bar{Y}\hat{Y}_t &= \bar{w}\bar{L}\hat{w}_t\hat{L}_t + \bar{r}\bar{K}\hat{r}_t\hat{K}_t \end{aligned}$$

   (c) Law of motion for capital: $\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \delta\hat{I}_t$

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + I_t \\ \bar{K}\left(1+\hat{K}_{t+1}\right) &= (1-\delta)\bar{K}\left(1+\hat{K}_t\right) + \bar{I}\left(1+\hat{I}_t\right) \\ \bar{K} + \bar{K}\hat{K}_{t+1} &= (1-\delta)\bar{K} + \bar{I} + (1-\delta)\bar{K}\hat{K}_t + \bar{I}\hat{I}_t \\ \bar{K} + \bar{K}\hat{K}_{t+1} &= \bar{K} + (1-\delta)\bar{K}\hat{K}_t + \delta\bar{K}\hat{I}_t \\ \bar{K}\hat{K}_{t+1} &= (1-\delta)\bar{K}\hat{K}_t + \delta\bar{K}\hat{I}_t \\ \hat{K}_{t+1} &= (1-\delta)\hat{K}_t + \delta\hat{I}_t \end{aligned}$$

   (d) Time resource constraint: $0 = \bar{L}\hat{L}_t + \bar{Z}\hat{Z}_t$

$$\begin{aligned} L_t + Z_t &= 1 \\ \bar{L}\left(1+\hat{L}_t\right) + \bar{Z}\left(1+\hat{Z}_t\right) &= 1 \\ \bar{Z} + \bar{L} + \bar{L}\hat{L}_t + \bar{Z}\hat{Z}_t &= 1 \\ 1 + \bar{L}\hat{L}_t + \bar{Z}\hat{Z}_t &= 1 \end{aligned}$$

2. Linear in logs

(a) Production Function: $\hat{Y}_t = \hat{\varepsilon}_{A,t} + \alpha \hat{K}_t + (1-\alpha)\hat{L}_t$

$$\begin{aligned}
Y_t &= \varepsilon_{A,t} K_t^{\alpha} L_t^{1-\alpha} \\
Y_t \left(\frac{\bar{Y}}{\bar{Y}}\right) &= \left(\frac{\bar{\varepsilon}_A}{\bar{\varepsilon}_A}\right) \varepsilon_{A,t} K_t^{\alpha} \left(\frac{\bar{K}}{\bar{K}}\right)^{\alpha} L_t^{1-\alpha} \left(\frac{\bar{L}}{\bar{L}}\right)^{1-\alpha} \\
\left(\frac{Y_t}{\bar{Y}}\right) \bar{Y} &= \left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right) \left(\frac{K_t}{\bar{K}}\right)^{\alpha} \left(\frac{L_t}{\bar{L}}\right)^{1-\alpha} \bar{\varepsilon}_A \bar{K}^{\alpha} \bar{L}^{1-\alpha} \\
&= \left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right) \left(\frac{K_t}{\bar{K}}\right)^{\alpha} \left(\frac{L_t}{\bar{L}}\right)^{1-\alpha} \bar{Y} \\
\left(\frac{Y_t}{\bar{Y}}\right) &= \left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right) \left(\frac{K_t}{\bar{K}}\right)^{\alpha} \left(\frac{L_t}{\bar{L}}\right)^{1-\alpha} \\
\ln\left(\frac{Y_t}{\bar{Y}}\right) &= \ln\left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right) + \alpha \ln\left(\frac{K_t}{\bar{K}}\right) + (1-\alpha)\ln\left(\frac{L_t}{\bar{L}}\right) \\
\hat{Y}_t &= \hat{\varepsilon}_{A,t} + \alpha \hat{K}_t + (1-\alpha)\hat{L}_t
\end{aligned}$$

(b) Law of motion for technology: $\hat{\varepsilon}_{A,t} = \rho \hat{\varepsilon}_{A,t-1} + u_t$

$$\begin{aligned}
\ln(\varepsilon_{A,t}) &= \rho \ln(\varepsilon_{A,t-1}) + u_t \\
\ln(\varepsilon_{A,t}) - \ln(\bar{\varepsilon}_A) &= \rho \ln(\varepsilon_{A,t-1}) - \ln(\bar{\varepsilon}_A) + u_t \\
\hat{\varepsilon}_{A,t} &= \rho \hat{\varepsilon}_{A,t-1} + u_t
\end{aligned}$$

(c) Labor supply decision:$\hat{w}_t = \hat{C}_t - \hat{Z}_t$

$$\begin{aligned}
L_t &= 1 - \frac{\sigma_L}{w_t} C_t \\
\frac{\sigma_L}{w_t} C_t &= 1 - L_t \\
\frac{\sigma_L}{w_t} C_t &= Z_t \\
\sigma_L C_t &= w_t Z_t \\
\left(\frac{C_t}{\bar{C}}\right) \sigma_L \bar{C} &= \left(\frac{w_t}{w}\right) \left(\frac{Z_t}{Z}\right) \bar{w} \bar{Z} \\
\left(\frac{C_t}{\bar{C}}\right) &= \left(\frac{w_t}{w}\right) \left(\frac{Z_t}{Z}\right) \\
\ln\left(\frac{C_t}{\bar{C}}\right) &= \ln\left(\frac{w_t}{w}\right) + \ln\left(\frac{Z_t}{Z}\right) \\
\hat{C}_t &= \hat{w}_t + \hat{Z}_t
\end{aligned}$$

(d) Wage rate and MPL:$\hat{w}_t = \hat{\varepsilon}_{A,t} + \alpha \hat{K}_t - \alpha \hat{L}_t$

$$\begin{aligned}
w_t &= (1-\alpha) \varepsilon_{A,t} K_t^{\alpha} L_t^{-\alpha} \\
w_t \left(\frac{\bar{w}}{\bar{w}}\right) &= (1-\alpha) \varepsilon_{A,t} \left(\frac{\bar{\varepsilon}_A}{\bar{\varepsilon}_A}\right) K_t^{\alpha} \left(\frac{\bar{K}}{\bar{K}}\right)^{\alpha} L_t^{-\alpha} \left(\frac{\bar{L}}{\bar{L}}\right)^{-\alpha} \\
\left(\frac{w_t}{\bar{w}}\right) \bar{w} &= \left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right) \left(\frac{K_t}{\bar{K}}\right)^{\alpha} \left(\frac{L_t}{\bar{L}}\right)^{-\alpha} (1-\alpha) \bar{\varepsilon}_A \bar{K}^{\alpha} \bar{L}^{-\alpha}
\end{aligned}$$

$$
\begin{aligned}
\left(\frac{w_t}{\bar{w}}\right) &= \left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right)\left(\frac{K_t}{\bar{K}}\right)^{\alpha}\left(\frac{L_t}{\bar{L}}\right)^{-\alpha} \\
\ln\left(\frac{w_t}{\bar{w}}\right) &= \ln\left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right) + \alpha\ln\left(\frac{K_t}{\bar{K}}\right) - \alpha\ln\left(\frac{L_t}{\bar{L}}\right) \\
\hat{w}_t &= \hat{\varepsilon}_{A,t} + \alpha\hat{K}_t - \alpha\hat{L}_t
\end{aligned}
$$

(e) Rental rate and MPK: $\hat{r}_t = \hat{\varepsilon}_{A,t} + (\alpha-1)\hat{K}_t + (1-\alpha)\hat{L}_t$

$$
\begin{aligned}
r_t &= \alpha\varepsilon_{A,t}K_t^{\alpha-1}L_t^{1-\alpha} \\
r_t\left(\frac{\bar{r}}{\bar{r}}\right) &= \alpha\varepsilon_{A,t}\left(\frac{\bar{\varepsilon}_A}{\bar{\varepsilon}_A}\right)K_t^{\alpha-1}\left(\frac{\bar{K}}{\bar{K}}\right)^{\alpha-1}L_t^{1-\alpha}\left(\frac{\bar{L}}{\bar{L}}\right)^{1-\alpha} \\
\left(\frac{r_t}{\bar{r}}\right)\bar{r} &= \alpha\left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right)\left(\frac{K_t}{\bar{K}}\right)^{\alpha-1}\left(\frac{L_t}{\bar{L}}\right)^{-\alpha}\bar{\varepsilon}_A\bar{K}^{\alpha-1}\bar{L}^{1-\alpha} \\
\left(\frac{r_t}{\bar{r}}\right) &= \left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right)\left(\frac{K_t}{\bar{K}}\right)^{\alpha-1}\left(\frac{L_t}{\bar{L}}\right)^{1-\alpha} \\
\ln\left(\frac{r_t}{\bar{r}}\right) &= \ln\left(\frac{\varepsilon_{A,t}}{\bar{\varepsilon}_A}\right) + (\alpha-1)\ln\left(\frac{K_t}{\bar{K}}\right) + (1-\alpha)\ln\left(\frac{L_t}{\bar{L}}\right) \\
\hat{r}_t &= \hat{\varepsilon}_{A,t} + (\alpha-1)\hat{K}_t + (1-\alpha)\hat{L}_t
\end{aligned}
$$

3. Non-linear

(a) Euler consumption equation:

$$
\begin{aligned}
1 &= \beta\mathbb{E}_t\left[\frac{C_t}{C_{t+1}}\left(r_{t+1} + (1-\delta)\right)\right] \\
&= \beta\mathbb{E}_t\left[\frac{C_t}{C_{t+1}}r_{t+1} + \frac{C_t}{C_{t+1}}(1-\delta)\right] \\
&= \beta\mathbb{E}_t\left[\frac{\bar{C}e^{\hat{C}_t}}{\bar{C}e^{\hat{C}_{t+1}}}\bar{r}e^{\hat{r}_{t+1}} + \frac{\bar{C}e^{\hat{C}_t}}{\bar{C}e^{\hat{C}_{t+1}}}(1-\delta)\right] \\
&= \beta\mathbb{E}_t\left[e^{\hat{C}_t-\hat{C}_{t+1}+\hat{r}_{t+1}}\bar{r} + e^{\hat{C}_t-\hat{C}_{t+1}}(1-\delta)\right] \\
&\approx \beta\left[\bar{r}\mathbb{E}_t\left(1+\hat{r}_{t+1}+\hat{C}_t-\hat{C}_{t-1}\right) + (1-\delta)\mathbb{E}_t\left(\hat{C}_t-\hat{C}_{t+1}\right)\right] \\
&= \beta\left[\bar{r}\mathbb{E}_t(1+\hat{r}_{t+1}) + (\bar{r}+(1-\delta))\mathbb{E}_t\left(\hat{C}_t-\hat{C}_{t+1}\right)\right] \\
&= \beta\bar{r}\mathbb{E}_t(1+\hat{r}_{t+1}) + \mathbb{E}_t\left(\hat{C}_t-\hat{C}_{t+1}\right) \\
\hat{C}_t &= \mathbb{E}_t\left(\hat{C}_{t+1}\right) - \beta\bar{r}\mathbb{E}_t(1+\hat{r}_{t+1})
\end{aligned}
$$

Where we used the fact that in steady state $\bar{r}-(1-\delta)=\beta^{-1}$.

Summarizing, the log-linearized system is given by:

$$
\begin{aligned}
\bar{Y}\hat{Y}_t &= \bar{C}\hat{C}_t + \bar{I}\hat{I}_t \\
\bar{Y}\hat{Y}_t &= \bar{w}\bar{L}\hat{w}_t\hat{L}_t + \bar{r}\bar{K}\hat{r}_t\hat{K}_t \\
\hat{K}_{t+1} &= (1-\delta)\hat{K}_t + \hat{I}_t\delta \\
0 &= \bar{L}\hat{L}_t + \bar{Z}\hat{Z}_t \\
\hat{Y}_t &= \hat{\varepsilon}_{A,t} + \alpha\hat{K}_t + (1-\alpha)\hat{L}_t \\
\hat{\varepsilon}_{A,t} &= \rho\hat{\varepsilon}_{A,t-1} + u_t \\
\hat{Z}_t &= \hat{C}_t - \hat{w}_t \\
\hat{w}_t &= \hat{\varepsilon}_{A,t} + \alpha\hat{K}_t - \alpha\hat{L}_t \\
\hat{r}_t &= \hat{\varepsilon}_{A,t} + (\alpha-1)\hat{K}_t + (1-\alpha)\hat{L}_t \\
\hat{C}_t &= \mathbb{E}_t\left(\hat{C}_{t+1}\right) - \beta\bar{r}\mathbb{E}_t\left(1+\hat{r}_{t+1}\right)
\end{aligned}
$$

Note that since they provide the same information, we can drop either the first or second of these equations. If we wanted to, we can also substitute out wages and the rental rate. The consensed log-linearized system is:

$$
\begin{aligned}
\bar{Y}\hat{Y}_t &= \bar{C}\hat{C}_t + \bar{I}\hat{I}_t \\
\hat{K}_{t+1} &= (1-\delta)\hat{K}_t + \hat{I}_t\delta \\
0 &= \bar{L}\hat{L}_t + \bar{Z}\hat{Z}_t \\
\hat{Y}_t &= \hat{\varepsilon}_{A,t} + \alpha\hat{K}_t + (1-\alpha)\hat{L}_t \\
\hat{\varepsilon}_{A,t} &= \rho\hat{\varepsilon}_{A,t-1} + u_t \\
\hat{Z}_t &= \hat{C}_t - \hat{\varepsilon}_{A,t} - \alpha\hat{K}_t + \alpha\hat{L}_t \\
\hat{C}_t &= \mathbb{E}_t\left(\hat{C}_{t+1}\right) - \beta\bar{r}\mathbb{E}_t\left(1+\hat{\varepsilon}_{A,t+1} + (\alpha-1)\hat{K}_{t+1} + (1-\alpha)\hat{L}_{t+1}\right)
\end{aligned}
$$

6 Estimating the model using Dynare

In this section we show how to estimate the model in MATLAB using Dynare through two methods. The first enters the models equations and automatically finds the steady state. The second enters the steady state and linearized equations.

6.1 Model

The preamble:

```
var Y C I K Leps_A;

varexo u;

parameters beta sigmaL delta alpha rho sigma_A;
```

The first line enters the parameters in the model; Output Y, consumption C, investment I, capital K, labor L, the wage rate w, the rental rate r and the aggregate productivity shock ε_A. The next line defines the exogenous variables; the error for the labor productivity shock u. Finally, the third row define the models parameters; β, σ_L, δ, α, ρ and σ_A.

Next, we calibrate the models parameters and define the steady state values:

beta = 0.99;

sigmaL = 1.75;

delta = 0.02;

alpha = 0.33;

sigma_A = 1;

rho = 0.95;

% Steady state

Lss = 0.3;

rss = 1/beta - 1 + delta;

Kss = Lss*(alpha/rss)^(1/(1-alpha));

Yss = Kss^alpha * Lss^(1-alpha);

Iss = delta*Kss;

Css = Yss - Iss;

Zss = 1 - Lss;

wss = epsilon*(1-alpha)*(Yss/Lss);

Next, we define the model which we want dynare to linearize. Since we want a log linearization we write each variable x_t as $\exp(x_t)$ (see the file "Introduction_to_Dynare.pdf" for more details):

model;

% Resource Constraints

exp(Y) = exp(C) + exp(I);

exp(1) = exp(L) + exp (Z);

% Household Equations

exp(C)^(-1) = beta*(((exp(C(+1)))^(-1))*(alpha*exp(eps_A(+1))*exp(K(+1))^(alpha-1)*exp(L(+1))^(1-alpha) + 1 - delta));

exp(Z) = (sigmaL/((1-alpha)*exp(eps_A)*exp(K)^(alpha)*exp(L)^(-alpha)))*exp(C);

exp(K(+1)) = (1-delta)*exp(K) + exp(I);

% Firms

exp(Y) = exp(eps_A)*(exp(K))^alpha*(exp(L))^(1-alpha);

% Shocks

eps_A = rho*eps_A(-1) + u;

end;

We also need to define some initial conditions for Dynare to start from when searching for the steady state. This also allows us to check our steady state calculations:

initval;

K = log(Kss);

C = log(Css);

L = log(Lss);

w = log(wss);

r = log(rss);

eps_A = 0;

end;

check;

steady;

Next, we define the shocks:

shocks;

var u = sigma_eps_L^2;

end;

Finally, we compute policy functions and IRF's:

stoch_simul(order=1,irf=100) Y C I K L Z;

Placing Y C I K L Z at the end provides an order for the IRFs.

6.2 Linearized Model

The preamble, calibration, shock specification and stochastic simulation are all the same as in the previous case. Thus, here we present the changes in the model specification and initial conditions:

model(linear);

% Resource Constraints

Yss*Y = Css*C + Iss*I;

Lss*L + Zss*Z = 0;

% Household Equations

C = C(+1) - beta*(1/beta - 1 + delta)*(eps_A(+1) + (alpha-1)*K(+1) + (1-alpha)*L(+1) + 1);

Z = C - eps_A - alpha*K + alpha*L;

K(+1) = (1-delta)*K + delta*I;

% Firms

Y = eps_A + alpha*K + (1-alpha)*L;

% Shocks

eps_A = rho*eps_A(-1) + u;

end;

Initial conditions:

initval;

K = Kss;

C = Css;

L = Lss;

I = Iss;

Z = Zss;

eps_A = 0;

end;

Note that the results are the same in either specification. **CURRENTLY FALSE?!**

References

[1]