# DSGE model

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## 1 The complete non-linearised model

Households

$$W_t C_t^{-\sigma} = \chi L_t^{\zeta} \tag{1}$$

$$1 = \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} R_t \tag{2}$$

$$C_t^{-\sigma} = \beta \frac{C_{t+1}^{-\sigma} R_t^n}{\pi_{t+1}} \tag{3}$$

Wholesalers

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{(1-\alpha)\Omega} \tag{4}$$

$$\frac{P_t^w}{P_t} = p_t^w = \frac{1}{\chi_t} \tag{5}$$

$$W_t = \frac{(1-\alpha)\Omega}{\chi_t} \frac{Y_t}{L_t} \tag{6}$$

$$W_t^e = \frac{(1-\alpha)(1-\Omega)}{\chi_t} Y_t \tag{7}$$

Capital producers

$$K_t = V(\frac{I_t}{K_{t-1}})K_{t-1} + (1-\delta)K_{t-1} \tag{8}$$

where $I_t$ is the investment in period $t$, $\delta$ is the rate of depreciation and

$$V(\frac{I_t}{K_{t-1}})K_{t-1} = I_t - \frac{\phi}{2}(\frac{I_t}{K_{t-1}} - \delta)^2 K_{t-1}$$

$$\frac{1}{Q_t} = 1 - \phi(\frac{I_t}{K_{t-1}} - \delta) \tag{9}$$

Retailers

$$1 = [\epsilon \pi_{t-1}^{1-\theta} + (1-\epsilon)(P_t^*)^{1-\theta}]^{\frac{1}{1-\theta}} \tag{10}$$

Monetary policy

$$log(R_t^n) - log(R^n) = \rho^{R^n}(log(R_{t-1}^n) - log(R^n)) + \rho^\pi \pi_{t-1} + \epsilon_t^{R^n} \tag{11}$$

Entrepreneurs

$$R_{t+1}^k = \frac{R_{t+1}^r + (1-\delta)Q_{t+1}}{Q_t} \tag{12}$$

$$N_{t+1} = \gamma(Q_t k_t R_{t+1}^k - (Q_t k_t - N_t)R_t - \mu Q_{t-1} K_t R_{t+1}^k \int_0^{\bar{x}_t} s\phi(s)ds) + W_{t+1}^e \tag{13}$$

$$C_t^e = (1-\gamma)(N_t - W_t^e) \tag{14}$$

$$\kappa = \frac{QK}{NW} \tag{15}$$

Market clearing

$$Y_t = C_t + I_t + G_t + C_t^e + \mu Q_{t-1} K_t R_t^k \int_0^{\bar{x}} s\phi(s)ds \tag{16}$$

Shocks

$$log(A_t) = \rho^A log(A_{t-1}) + \epsilon_t^A \tag{17}$$

$$log(\frac{G_t}{Y_t}) = \rho^G log(\frac{G_{t-1}}{Y_{t-1}}) + \epsilon_t^G \tag{18}$$

$$log(\sigma_{s,t}) = \rho^{\sigma_s} log(\sigma_{s,t-1}) + \epsilon_t^{\sigma_s} \tag{19}$$

Optimality conditions

$$E_t[\kappa_t^{-p} {R^k_{t+1}}^{1-p} g_{t+1} + \lambda_{t+1}[R^k_{t+1} h_{t+1} - R_t]] = 0 \tag{20}$$

$$\frac{(\kappa_t R^k_{t+1})^{1-p} g_{\bar{x},t+1}}{1-p} + \lambda_{t+1} \kappa_t R^k_{t+1} h_{\bar{x},t+1} = 0 \tag{21}$$

$$\frac{(\kappa_t R^k_{t+1})^{1-p} g_{\underline{x},t+1}}{1-p} + \lambda_{t+1} \kappa_t R^k_{t+1} h_{\underline{x},t+1} = 0 \tag{22}$$

$$\frac{(\kappa_t R^k_{t+1})^{1-p} g_{\bar{R},t+1}}{1-p} + \lambda_{t+1} \kappa_t R^k_{t+1} h_{\bar{R},t+1} = 0 \tag{23}$$

Where

$$\begin{aligned} h(\underline{x}_{t+1}, \bar{x}_{t+1}, \bar{R}_{t+1}, \sigma_{s,t}) = (1-\mu) \int_{\underline{x}_{t+1}}^{\bar{x}_{t+1}} s\phi(s)ds + \bar{R} \int_{\bar{x}_{t+1}}^{\infty} \phi(s)ds - \eta \int_{\underline{x}_{t+1}}^{\bar{x}_{t+1}} s\phi(s)ds \\ - \underline{x}_{t+1} \int_{\eta s_{t+1}}^{s_{t+1}} \phi(s)ds - \mu \int_0^{\underline{x}_{t+1}} s\phi(s)ds \end{aligned}$$

and

$$\begin{aligned} g(\underline{x}_{t+1}, \bar{x}_{t+1}, \bar{R}_{t+1}, \sigma_{s,t}) = \int_0^{\underline{x}_{t+1}} s^{1-p}\phi(s)ds + {\underline{x}_{t+1}}^{1-p} \int_{\eta s_{t+1}}^{s_{t+1}} \phi(s)ds \\ + \eta^{1-p} \int_{\underline{x}_{t+1}}^{\bar{x}_{t+1}} s^{1-p}\phi(s)ds + \int_{\bar{x}_{t+1}}^{\infty} (s-\bar{R})^{1-p}\phi(s)ds \end{aligned}$$

are, respectively, the lender and entrepreneurs returns and $log(s^i_{t+1}) \rightsquigarrow N(-\frac{\sigma^2_{s,t}}{2}, \sigma^2_{s,t})$.
Entrepreneurs return

$$E_t[\frac{(\kappa_t R^k_{t+1})^{1-p} g(\underline{x}_{t+1}, \bar{x}_{t+1}, \bar{R}_{t+1}, \sigma_{s,t})}{1-p}] \tag{24}$$

Lender participation constraint

$$\kappa_t R^k_{t+1} h(\underline{x}_{t+1}, \bar{x}_{t+1}, \bar{R}_{t+1}, \sigma_{s,t}) = (\kappa_t - 1) R_t \tag{25}$$

# 2 The complete log-linearised model

The log-linear model has 18 equations and 18 variables. Let lower variable denote percent deviations from the steady state ($c_t = \hat{C}_t = \frac{C_t - \bar{C}}{\bar{C}}$), and let capital letter without time subscript denotes the steady state value of the respective variable. As in BGG, we find it convenient to express the complete log-linearised model in terms of blocks of equations:

**•Aggregate Demand**

$$-\sigma(c_t - c_{t+1}) = r_t \tag{26}$$

$$y_t = \frac{C}{Y}c_t + \frac{G}{Y}g_t + \frac{I}{Y}i_t + \frac{C^e}{Y}c_t^e \tag{27}$$

$$\hat{\kappa}_t = \nu_p(E_t r_{t+1}^k - r_t) + \nu_\sigma \hat{\sigma}_{s,t} \tag{28}$$

$$c_t^e = (1-\gamma)(\frac{N}{C^e}n_t - \frac{W^e}{C^e}w_t^e) \tag{29}$$

$$q_t = \delta\phi(i_t - k_{t-1}) \tag{30}$$

$$r_{t+1}^k = (1-\varrho)(y_{t+1} - x_{t+1} - k_t) + \varrho q_{t+1} - q_t \tag{31}$$

$$\kappa_t + n_t = q_t + k_t \tag{32}$$

$$r_t^n = r_t + E_t\hat{\pi}_{t+1} \tag{33}$$

**•Aggregate Supply**

$$y_t = a_t + \alpha k_{t-1} + \Omega(1-\alpha)l_t \tag{34}$$

$$(\zeta+1)l_t + x_t + \sigma c_t = y_t \tag{35}$$

$$y_t = x_t + w_t^e \tag{36}$$

$$\hat{\pi}_t = -\frac{(1-\epsilon)(1-\epsilon\beta)}{\epsilon}x_t + \beta E_t\hat{\pi}_{t+1} \tag{37}$$

**•Evolution of state variables**

$$k_t = \delta i_t + (1-\delta)k_{t-1} \tag{38}$$

$$n_{t+1} = \gamma R \kappa n_t (\kappa_t (\frac{R^k}{R} r^k_{t+1} - r_t) + \frac{1}{\kappa} r_t) + \frac{W^e}{N} w^e_{t+1} \tag{39}$$

**•Monetary policy rule**

$$r^n_t = \rho^{R^n} r^n_{t-1} + \rho^{\pi} \pi_{t-1} + \epsilon^{R^n}_t \tag{40}$$

**•Shock processes**

$$a_t = \rho^A a_{t-1} + \epsilon^A_t \tag{41}$$

$$g_t = \rho^G g_{t-1} + \epsilon^G_t \tag{42}$$

$$\sigma_{s,t} = \rho^{\sigma_s} \sigma_{s,t-1} + \epsilon^{\sigma_s}_t \tag{43}$$