1 Representative agent

1.1 Households

Time is discrete and continues forever. There is a unit continuum of representative households, whose lifetime preference is

$$U(c,l) = \sum_{t=0}^{\infty} \Pi_{s=0}^{s=t} b_s (log(c_t) - \eta * \frac{l_t^{\chi}}{1+\chi})$$

where c_t and l_t denote household consumption and labor in period t.

Households can borrow and lend in a default-free one-period nominal bond market at the continuously-compounded interest rate i_t. The use of continuous compounding simplifies the bond-pricing equations below. Each period, the household faces a flow budget constraint

$$P_t c_t + e^{-i_t} B_{t+1}^{(1)} = B_t^{(1)} + w_t l_t + d_t$$

where $B_t^{(1)}$ denotes beginning-of-period holdings of nominal one period bond and w_t and d_t denote the nominal wage and exogenous transfers to the household, respectively.

The inter-temporal preference shock b_t follows :

$$b_t = b + \varepsilon_t^m \tag{1}$$

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$\begin{aligned} \frac{\partial U(B_t^{(1)};\Theta_t)}{\partial c_t} &= \Lambda_t \\ \frac{\partial U(B_t^{(1)};\Theta_t)}{\partial l_t} &= \Lambda_t \frac{w_t}{P_t} \\ e^{i_t} \mathrm{E}_t sdf_{t+1} \frac{P_t}{P_{t+1}} &= 1 \end{aligned}$$

where Λ_t denotes the Lagrange multiplier on the household budget constraint, and sdf_t is household's stochastic discount factor given by

$$sdf_t \equiv b_t \frac{c_{t-1}}{c_t}$$

1.2 Firms

The economy also contains a continuum of infintely-lived monopolistically competitive firms indexed by $s \in [0,1]$, each producing a single differentiated good. Firms hire labor from households in a competitive market and have identical Cobb-Douglas production functions,

$$y_t(s) = k^{1-\theta} l_t(s)^{\theta}$$

The output of each firm s is purchased by a perfectly competitive final goods sector, which aggregates the differentiated goods into a single final good using a CES production technology,

$$Y_t = [\int_0^1 y_t(s)^{1/\lambda} df]^{\lambda}$$

Each intermediate firm s thus faces a downward-sloping demand curve for its product with elasticity $\lambda/(1-\lambda)$.

$$y_t(s) = (\frac{p_t(s)}{P_t})^{-\lambda/(\lambda-1)} Y_t$$

where Y_t denotes the quantity of the final good and $\lambda > 1$ is a parameter. P_t is the CES aggregate price of the final good,

$$P_t = [\int_0^1 p_t(s)^{1/1-\lambda} df]^{1-\lambda}$$

Firms set prices optimally subject to nominal rigidities in the form of Calvo (1983). The standard New Keynesian price optimality condition is

$$p_t^*(s) = \lambda \frac{\mathrm{E}_t \sum_{j=0}^{\infty} sdf_{t,t+j}(P_t/P_{t+j})\xi^j y_{t+j}(s) MC_{t+j}(s)}{\mathrm{E}_t \sum_{j=0}^{\infty} sdf_{t,t+j}(P_t/P_{t+j})\xi^j y_{t+j}(s) e^{j\bar{\pi}}}$$

where $MC_t(s)$ denotes the (nominal) marginal cost for firm s at time t,

$$MC_t(s) = \frac{w_t l_t(s)}{\theta y_t(s)}$$

1.3 Aggregate Economy

Let L_t denote the aggregate quantity of labor demanded by firms,

$$L_t = \int_0^1 l_t(s) ds$$

Labor market equilibrium requires that

$$L_t = l_t$$

And L_t satisfies

$$Y_t = K^{1-\theta} L_t^{\theta} / \triangle_t$$

where $K = k$ denotes the aggregate capital stock and the price dispersion $\triangle_t$.

Equilibrium in the final goods market requires

$$Y_t = C_t$$

where $C_t = c_t$ denotes aggregate consumption demanded by households.

1.4 Central Bank

Finally, there is a monetary authority that sets the one-period nominal interest rate i_t according to the following policy rule,

$$i_t = r + \pi_t + \phi_\pi(\pi_t - \bar{\pi}) + \phi_y(y_t - y_{ss}) \tag{2}$$

2 Two-agents

Only a fraction w of the households will be hit by ε_t^m, while $1-w$ of the households has constant discount factor.