Advanced Macroeconomics

Chapter V:
New Keynesian Theory

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Outline

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Setup

The setup is as in the classical monetary economy, with the additional features

- **Monopolistic competition.** Instead of an Walrasian auctioneer clearing all markets, prices of goods and inputs are set by economic agents. The assumption of perfect competition is abandoned.
- **Nominal rigidities.** This is the point why the models are called new Keynesian. As Keynes, new Keynesians assume prices and/or wages to not be fully flexible, i.e. **sticky**. You will see that this has important effects.

Households

- The representative HH still maximizes the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right\}. \tag{1}$$

- Consumption does not consist of a single good anymore, but is a bundle of infinitely many goods, or **varieties** (Dixit and Stiglitz, 1977, CES-aggregator):

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \qquad \epsilon > 1, \tag{2}$$

  where i denotes the specific consumption good, and ϵ is the constant **elasticity of substitution (CES) between these goods**.

- We also have the solvency condition $\lim_{T\to\infty} E_t\{B_T\} \geq 0$ and the period BC stating for each single variety $C_t(i)$ you have to pay its price $P_t(i)$:

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t L_t - T_t. \tag{3}$$

Firms

- There are infinitely many firms in the economy, each producing exactly one **different variety**, but with an identical production function
- Firm producing good i – itself also denoted by the index i – produces according to

$$Y_t(i) = A_t L_t(i), \tag{4}$$

  where the technology level A_t is common to all firms
- We still have constant returns to scale, i.e. α from equation (**??**) is 0
- We assume something new here: firms **set their own prices**
- Until now, we had perfect competition and a Walrasian auctioneer, resulting in $P_t = MC_t$
- We needed this mechanism because firms faced zero demand with a too high price, and the entire economy-wide demand with a too low price

Deriving Optimal Behavior: The FOCs

- The FOCs are going to be a bit more complicated than usual: The HH has to decide how much to consume in total and how much to consume of each variety
- Based on this decision, i.e. a **demand schedule for each variety**, the firms have to decide on which price to set
- The existence of a **downward sloping demand curve**, due to the way how the consumption bundle is composed, is the reason for monopolistic competition
- However, firms won't be free to set prices
- But first we look at the implications of monopolistic competition.

Household Consumption Decision

- The problem of the HHs can be decomposed into a two-stage problem
  1. decide on how much to buy of each variety, given total consumption expenditure
  2. determine this total consumption expenditure
- **Demand for each $C(i)$**
  Maximize the consumption bundle, s.t. the restriction that you spend your total consumption expenditure M (which is determined later):

$$\mathcal{L} = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} - \lambda_t \left(\int_0^1 P_t(i) C_t(i) di - M \right). \tag{5}$$

- The derivative w.r.t. to a specific $C_t(i)$ is

$$\frac{\partial \mathcal{L}}{\partial C_t(i)} = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}-1} C_t(i)^{\frac{\epsilon-1}{\epsilon}-1} - \lambda_t P_t(i) \overset{!}{=} 0. \tag{6}$$

Searching for Lambda

- The first bracket can be rewritten with the definition of the basket (2):

$$C_t^{\frac{1}{\epsilon}} C_t(i)^{-\frac{1}{\epsilon}} - \lambda_t P_t(i) \equiv 0 \Rightarrow C_t(i) = (\lambda_t P_t\,(i))^{-\epsilon} C_t \qquad (7)$$

- But what is λ_t? There are at least two ways how to get this
- Start with the mathematical one: Insert the last equation into the bundle definition (2)

$$C_t = \left(\int_0^1 (\lambda_t P_t(i))^{1-\epsilon} C_t^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$\Rightarrow \lambda_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{\epsilon-1}} \qquad (8)$$

$$
\begin{aligned}
C_t &= \left(\int_0^1 (\lambda_t P_t(i))^{1-\epsilon} C_t^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \\
C_t &= \lambda_t^{-\epsilon} \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}} C_t \\
\lambda_t &= \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{\epsilon-1}} \qquad (9)
\end{aligned}
$$

Definition of the Price Index

- Let us now define a price index P_t that makes notation easier
- This P_t should be such that we can write total consumption expenditures M as:

$$\int_0^1 P_t(i) C_t(i) di \equiv P_t C_t. \tag{10}$$

- This definition has to be consistent with our FOC for $C_t(i)$. Hence, insert equation (7):

$$\int_0^1 P_t(i)^{1-\epsilon} \lambda_t^{-\epsilon} C_t di = P_t C_t$$

$$\int_0^1 P_t(i)^{1-\epsilon} di = P_t \lambda_t^{\epsilon}. \tag{11}$$

The Price Index

- Now, replace λ_t in (11) with (8) to get

$$\int_0^1 P_t(i)^{1-\epsilon} di = P_t \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$\left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} = P_t. \tag{12}$$

- This is the definition of our price index P_t. Do you notice something?
- It's actually the inverse of lambda:

$$\lambda_t = P_t^{-1}. \tag{13}$$

- Intuitive solution: remember interpretation of λ_t as shadow price
- By how much would C_t (the objective function) increase, if we ease the constraint marginally (i.e. increase M)?
- M buys you M/P bundles and one more € buys you $1/P$ bundles

Demand Function for Variety i

- Using this fact in the FOC for $C_t(i)$ (7) yields

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t, \tag{14}$$

which is the demand function for good $C_t(i)$.

- Notice the differences to perfect competition:
  - If $P_t(i) > P_t$ the resulting demand is not zero, and if $P_t(i) < P_t$, the resulting demand is not equal to C_t.
  - The consumer always spends at least something on each variety. This is called **love of variety**.
  - To which extend the demand reacts to price changes depends on the **elasticity of substitution** ϵ.
  - Because firms can increase prices without facing zero demand, this situation is called **monopolistic competition**. The reason is that goods are **imperfect substitutes** in the UF

Second Stage: Finding Total Consumption

- Having found the demand for each variety i given total consumption C_t, we now turn to finding this total consumption
- With the definition of the price index (10) we can write the BC (3) as

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t L_t - T_t. \tag{15}$$

- This is the same as in the CLM, equation (**??**)
- Hence, solving for C_t can be done as always before, without worrying that C_t is actually made up of many goods.
- The resulting FOCs are

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \qquad \text{(??')}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\varphi} \qquad \text{(??')}$$

Firms

- Firms still maximize profits, but take the demand into account
- Assuming that they have to satisfy their entire demand once they have announced their price (i.e. $Y_t(i) = C_t(i)$), a possible Lagrangian – with labor already substituted out by the PF (4) – looks like

$$\max_{Y_t(i),P_t(i),\lambda_t(i)} \mathcal{L} = \max_{Y_t(i),P_t(i),\lambda_t(i)} \left\{ \begin{array}{l} P_t(i) Y_t(i) - W_t \frac{Y_t(i)}{A_t} \\ -\lambda_t(i) \left(Y_t(i) - \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \right) \end{array} \right\} \tag{16}$$

- Firms now optimize over their price and their output (labor supply), taking the constraint into account (λ).
- Although $Y_t(i)$ is determined by the price, the derivative of $\mathcal{L}$ w.r.t. both must be taken for the firm to take this into account
- Each firm is extremely small and does therefore not take its own impact on the aggregate variables P_t and W_t into account (i.e. treats those as constants)

Optimal Price I

- *Exercise 1:* Show that the resulting optimal price is

$$P_t(i) = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{A_t} \,, \tag{17}$$

- Note that

$$\frac{\partial \Psi_t}{\partial Y_t(i)} = \frac{W_t}{A_t} \equiv \xi_t \tag{18}$$

are just the **nominal marginal costs**. i.e. the derivative of nominal costs

$$\Psi_t \equiv W_t \frac{Y_t(i)}{A_t} \tag{19}$$

in the Lagrangian w.r.t. $Y_t(i)$.

- In the following, denote **real marginal costs** of period t as

$$MC_t = \frac{W_t}{P_t A_t} = \frac{\xi_t}{P_t}. \tag{20}$$

$$\frac{\partial L}{\partial Y_t(i)} = P_t(i) - \frac{W_t}{A_t} - \lambda_t(i) = 0 \Rightarrow P_t\left(i\right) - \frac{W_t}{A_t} = \lambda_t\left(i\right)$$

$$\frac{\partial L}{\partial P_t\left(i\right)} = Y_t\left(i\right) + \lambda_t\left(i\right) C_t\left(-\epsilon\right)\left(\frac{1}{P_t}\right)^{-\epsilon} P_t(i)^{-\epsilon-1} = 0$$

$$\Rightarrow Y_t\left(i\right) = \lambda_t\left(i\right) C_t\left(\frac{P_t\left(i\right)}{P_t}\right)^{-\epsilon} \frac{1}{P_t\left(i\right)}\epsilon$$

$$\frac{\partial L}{\partial \lambda(i)} \Rightarrow Y_t\left(i\right) = C_t\left(\frac{P_t\left(i\right)}{P_t}\right)^{-\epsilon}$$

$$Y_t\left(i\right) = \lambda_t\left(i\right) C_t\left(\frac{P_t\left(i\right)}{P_t}\right)^{-\epsilon} \frac{1}{P_t\left(i\right)}\epsilon = \left(P_t\left(i\right) - \frac{W_t}{A_t}\right) Y_t\left(i\right) \frac{1}{P_t\left(i\right)}\epsilon$$

$$\Rightarrow P_t(i) = \left(P_t(i) - \frac{W_t}{A_t}\right)\epsilon$$

$$\Rightarrow P_t(i) = \frac{\epsilon}{\epsilon - 1}\frac{W_t}{A_t}$$

Optimal Price II

- We have the standard outcome of monopolistic competition that the **price is a markup over marginal costs**:

$$P_t(i) = \mathcal{M} \cdot \xi_t. \tag{21}$$

- The markup $\mathcal{M} = \frac{\epsilon}{\epsilon - 1}$ is a function of the elasticity of substitution
- If $\epsilon \to \infty$ (i.e. perfect competition), then $\mathcal{M} \to 1$ (as on slide **??**). The smaller ϵ, the larger is the markup
- Because of constant returns to scale ($\alpha = 0$), marginal costs are independent of production, and therefore the same for all firms

Optimal Price Setting

- The price setting problem is changed substantially by this modification; formerly intratemporal problem becomes **dynamic**
- The firm must take future and past into account
- If a firm cannot change the price tomorrow, today's price matters for tomorrow's profits. The probability of not being able to reset is θ
- Let's consider one period after the other. Profits today of firm i are:

$$\Omega_t^i(P_t^i) = P_t^i Y_t^i - \Psi_t(Y_t^i), \tag{22}$$

where

- P_t^i is today's price of firm i (short for $P_t(i)$)
- Y_t^i is today's sales
- $\Psi_t(\bullet)$ is the nominal cost function (equation (19))

Tomorrow's Profits

- Tomorrow's expected, discounted profits are

$$\Omega^i_{t+1}(P^i_{t+1}) = E_t\{Q_{t,t+1}(P^i_{t+1}Y^i_{t+1} - \Psi_{t+1}(Y^i_{t+1}))\} \tag{23}$$

where

$$Q_{t,t+1} = \beta(C_{t+1}/C_t)^{-\sigma}(P_t/P_{t+1}) \tag{24}$$

is the **stochastic discount factor** (SDF) between periods t and $t+1$

- The SDF equals the price of a bond that pays one unit of money tomorrow (see equation (**??**). The intuition is straightforward: if one € tomorrow (the bond) costs Q €s today, this means that x €s are valued xQ €s today
- Underlying idea: firms are owned by HH and use their SDF to value cash flows
- What is P^i_{t+1}? If the firm can reset its price (probability $(1-\theta)$), then it will be a new, optimal price that is independent of the past. In this case, the firm would not have to bother today about P^i_{t+1}

Tomorrow's Implications for Today

- However, with probability θ the firm cannot reset its price tomorrow and still charges P_t^i
- In this case, tomorrow's expected, discounted profits will be

$$\Omega_{t+1}^i\left(P_t^i\right) = E_t\left\{Q_{t,t+1}\left(P_t^i Y_{t+1|t}^i - \Psi_{t+1}\left(Y_{t+1|t}^i\right)\right)\right\} \tag{25}$$

- $Y_{t|t+1}^i$ is the amount of goods sold in $t+1$ if charging P_t^i, i.e. the sales of a firm that last reset its price in t:

$$Y_{t+1|t}^i = \left(\frac{P_t^i}{P_{t+1}}\right)^{-\epsilon} C_{t+1} \tag{26}$$

- P_t^i affects Ω_t^i for sure, and with the probability θ also Ω_{t+1}^i
- The sum of expected profits today and tomorrow *that depend on today's price* becomes

$$\begin{aligned}\Omega_t^i\left(P_t^i\right) + \theta\Omega_{t+1}^i\left(P_t^i\right) = & P_t^i Y_t^i - \Psi_t\left(Y_t^i\right) \\ & + \theta E_t\left\{Q_{t,t+1}\left(P_t^i Y_{t+1|t}^i - \Psi_{t+1}\left(Y_{t+1|t}^i\right)\right)\right\}\end{aligned} \tag{27}$$

Firm's General Problem

- Firms maximize the expected sum of future profits that depend on the optimal price set today (not to 2 but infinitely many periods), denoted by P_t^*:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t}^i - \Psi_{t+k} \left(Y_{t+k|t}^i \right) \right) \right\}, \qquad (28)$$

with

$$Q_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma} (P_t/P_{t+k}), \qquad (29)$$

$$\Psi_{t+k} \left(Y_{t+k|t}^i \right) \overset{(18)+(19)}{=} \xi_{t+k} Y_{t+k|t}^i, \qquad (30)$$

and subject to the sequence of demand constraints

$$Y_{t+k|t}^i = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \quad \forall \quad k \geq 0. \qquad (31)$$

Firm's First-Order Condition

- *Exercise 2:* Show that the FOC for this problem is

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y^i_{t+k|t} (P^*_t - \mathcal{M} \xi_{t+k}) \right\} = 0, \tag{32}$$

where $\xi_{t+k} = \partial \Psi_{t+k|t}(Y^i_{t+k|t}) / \partial Y^i_{t+k|t}$ are (nominal) marginal costs in period $t+k$, which are the same for all firms

- The markup is $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$
- *Exercise 3:* Show that in case of no price rigidities, $\theta = 0$, the optimal price again collapses to

$$P^*_t = \mathcal{M} \xi_t \ , \tag{33}$$

i.e. implies that the firm's problem is again purely intratemporal

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t}^i - \Psi_{t+k} \left(Y_{t+k|t}^i \right) \right) \right\}$$

$$s.t. Y_{t+k|t}^i = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$$

$$\Psi_{t+k} \left(Y_{t+k|t}^i \right) = \xi_{t+k} Y_{t+k|t}^i$$

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left(\left(P_t^* \right)^{1-\epsilon} \left(\frac{1}{P_{t+k}} \right)^{-\epsilon} C_{t+k} - \left(P_t^* \right)^{-\epsilon} \left(\frac{1}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \xi_{t+k} \right) \right\}$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left((1-\epsilon) \left(P_t^* \right)^{-\epsilon} \left(\frac{1}{P_{t+k}} \right)^{-\epsilon} C_{t+k} - (-\epsilon) \left(P_t^* \right)^{-\epsilon-1} \left(\frac{1}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \xi_{t+k} \right) \right\} =$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} - \frac{\epsilon}{\epsilon - 1} \frac{1}{P_t^*} \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \xi_{t+k} \right) \right\} = 0$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t}^i \left(P_t^* - \mathcal{M} \xi_{t+k} \right) \right\} = 0$$

$$\sum_{k=0}^{\infty} 0^k E_t \left\{ Q_{t,t+k} Y^i_{t+k|t} \left(P^*_t - \mathcal{M} \xi_{t+k} \right) \right\} = 0$$

$$Q_{t,t} Y^i_{t|t} \left(P^*_t - \mathcal{M} \xi_t \right) = 0$$

$$P^*_t = \mathcal{M} \xi_t$$

Getting a Unique Steady State

- Unfortunately, from here on it is not so easy. The best way is to first linearize the above mentioned FOC for firms.
- Our linearizations are only valid if we are close to the steady state. But for this we need a **well-defined steady state**!
- The price level for example is in general not determined. Hence, we don't know P in steady state, but we **assume zero steady state inflation**: $P_t^{SS}/P_{t-1}^{SS} = 1$
- Hence, we divide equation (32) by P_{t-1} and split $Q_{t,t+k}$ into the deterministic, non-stationary part β^k and the part that is 1 in steady state:

$$(C_{t+k}/C_t)^{-\sigma}(P_t/P_{t+k}) \equiv \Lambda_{t,t+k} \tag{34}$$

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$$\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left\{\Lambda_{t,t+k}Y_{t+k|t}^i\left(\frac{P_t^*}{P_{t-1}} - \mathcal{M}MC_{t+k}\frac{P_{t+k}}{P_{t-1}}\right)\right\} = 0, \tag{35}$$

where $MC_{t+k} \equiv \xi_{t+k}/P_{t+k}$ are **real marginal costs** in period $t+k$.

Steady State

- In our steady state, we assume zero inflation, i.e. gross inflation $\frac{P_{t+1}^{SS}}{P_t^{SS}} = 1$
- Furthermore, the individual prices are the same as the price index, such that $\frac{P_t^*}{P_{t-1}} = 1$
- Therefore, real marginal costs $MC_{t+k} = MC$ and output $Y_{t+k|t}^i = Y^i$ are constant over time, even if the firm didn't reset its price for a long time
- *Exercise 4:* Show that in steady state

$$MC = \frac{\epsilon - 1}{\epsilon} \tag{36}$$

- Equation (35) in steady state is therefore

$$\sum_{k=0}^{\infty} (\theta\beta)^k \{Y (1 - \mathcal{M}MC)\} = 0, \tag{37}$$

which implies for each period (each part of the sum) that $MC = 1/\mathcal{M}$.

From (17) and (18):

$$P_t(i) = \frac{}{-1} \frac{W_t}{A_t}$$

$$P_t = \mathcal{M}\xi$$

$$MC = \frac{1}{\mathcal{M}}$$

Linearizing the Optimal Price I

- Let's first split the sum in equation (35):

$$\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left\{\Lambda_{t,t+k}Y^i_{t+k|t}\frac{P^*_t}{P_{t-1}}\right\}$$
$$=\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left\{\Lambda_{t,t+k}Y^i_{t+k|t}\mathcal{M}MC_{t+k}\frac{P_{t+k}}{P_{t-1}}\right\}. \tag{38}$$

- *Exercise 5:* Show that the LHS can be linearized as:

$$LHS \approx \sum_{k=0}^{\infty}(\theta\beta)^k(\hat{p}^*_t-\hat{p}_{t-1})Y^i+\sum_{k=0}^{\infty}(\theta\beta)^k(\hat{\Lambda}_{t,t+k}+\hat{y}^i_{t+k|t})Y^i. \tag{39}$$

- Since prices don't depend on k, we have

$$\sum_{k=0}^{\infty}(\theta\beta)^k(\hat{p}^*_t-\hat{p}_{t-1})=(\hat{p}^*_t-\hat{p}_{t-1})\sum_{k=0}^{\infty}(\theta\beta)^k=\frac{\hat{p}^*_t-\hat{p}_{t-1}}{1-\theta\beta} \tag{40}$$

Linearizing the Optimal Price II

- Let's first split the sum in equation (35):

$$\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left\{\Lambda_{t,t+k}Y^i_{t+k|t}\frac{P^*_t}{P_{t-1}}\right\}$$
$$=\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left\{\Lambda_{t,t+k}Y^i_{t+k|t}\mathcal{M}MC_{t+k}\frac{P_{t+k}}{P_{t-1}}\right\}. \quad (38)$$

- *Exercise 6:* Show that the RHS can be linearized as:

$$RHS \approx \sum_{k=0}^{\infty}(\theta\beta)^k(\hat{\Lambda}_{t,t+k}+\hat{y}^i_{t+k}+\widehat{mc}_{t+k}+\hat{p}_{t+k}-\hat{p}_{t-1})Y^i. \quad (41)$$

- If we combine the LHS and the RHS, $\hat{\Lambda}$ and $\hat{y}^i$ cancel for each part of the sum
- p_{t-1} can also be canceled, but we leave it for the moment

Linearizing the Optimal Price III

- Combining (39) and (41):

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\widehat{mc}_{t+k} + \hat{p}_{t+k} - \hat{p}_{t-1}\} \tag{42}$$

- To get some intuition, note that $\widehat{mc}_{t+k}$ is the percentage deviation of MC_{t+k} from its steady-state value $1/\mathcal{M}$, or in logs:

$$\widehat{mc}_{t+k} = \log MC_{t+k} - \log(1/\mathcal{M}) = \log MC_{t+k} + \log(\mathcal{M}) \tag{43}$$

- Inserting this into (42) yields

$$\hat{p}_t^* = \log(\mathcal{M}) + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\log MC_{t+k} + \hat{p}_{t+k}\}, \tag{44}$$

  i.e. a firm that resets its price will chose a **markup over a weighted average of current and expected *nominal* marginal costs**

- The weight of future marginal costs depends on the discount factor and the probability that today's price will be still effective in the respective period

Aggregate Price Level Dynamics

- A fraction $(1-\theta)$ of all firms can reset their prices at a given period. As we see in (42), they will all set the same price, since they face an identical problem
- Let $S(t) \subset [0,1]$ be the set of firms that cannot set a new price in period t. Then we get from (12)

$$P_t = \left[\int_{S(t)} P_{t-1}^{1-\epsilon}(i)di + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$= \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} . \quad (45)$$

- The last line stems from the fact that the distribution of prices among the random sample of firms not adjusting in period t is the same as the distribution of effective prices in period $t-1$, with total mass reduced to θ

Linearizing the Price Level

- To make things stationary, we divide both sides by P_{t-1} and then raise both sides to the power of $1-\epsilon$,

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}. \tag{46}$$

- In steady state $\Pi = \frac{P_t^{SS}}{P_{t-1}^{SS}} = 1 = \frac{P_t^*}{P_{t-1}^{SS}}$.
- *Exercise 7:* Show that a linearization around this steady state yields

$$\pi_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1}), \tag{47}$$

- Remember: π_t is the percentage deviation from steady-state gross inflation (=1), which equals net inflation. If inflation is 3%, then $\Pi = 1.03$ and $\pi = 0.03$.

A Short Representation

- *Exercise 8:* Show that the equation for the optimal price (42) can be recursively written to imply

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\widehat{mc}_{t+k} + \hat{p}_{t+k} - \hat{p}_{t-1}\} \quad (48)$$

$$= (1-\beta\theta)\widehat{mc}_t + \pi_t + \beta\theta E_t\{\hat{p}_{t+1}^* - \hat{p}_t\}. \quad (49)$$

- Plugging the movement of the price level (47) into this equation, we get

$$\boldsymbol{\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda\widehat{mc}_t}, \quad (50)$$

with

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}. \quad (51)$$

- Important: λ is not a Lagrange multiplier

$$= (1-\beta\theta)\left(\widehat{mc}_t + \hat{p}_t - \hat{p}_{t-1}\right) + (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left\{\widehat{mc}_{t+k} + \hat{p}_{t+k} - \hat{p}_{t-1}\right\}$$

$$= (1-\beta\theta)\left(\widehat{mc}_t + \hat{p}_t - \hat{p}_{t-1}\right) + (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left\{\widehat{mc}_{t+k} + \hat{p}_{t+k}\right\}$$

$$-(1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left\{\hat{p}_{t-1}\right\}$$

$$+(1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left\{\hat{p}_t\right\} - (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left\{\hat{p}_t\right\}$$

$$= (1-\beta\theta)\left(\widehat{mc}_t + \hat{p}_t - \hat{p}_{t-1}\right) + (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left\{\widehat{mc}_{t+k} + \hat{p}_{t+k}\right\}$$

$$-(1-\beta\theta)\,\beta\theta\hat{p}_{t-1}\sum_{k=0}^{\infty}(\beta\theta)^k$$

$$+(1-\beta\theta)\,\beta\theta\hat{p}_t\sum_{k=0}^{\infty}(\beta\theta)^k - (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left\{\hat{p}_t\right\}$$

$$= (1-\beta\theta)\left(\widehat{mc}_t + \hat{p}_t - \hat{p}_{t-1}\right) + (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left\{\widehat{mc}_{t+k} + \hat{p}_{t+k} - \hat{p}_t\right\}$$

$$-(1-\beta\theta)\,\hat{p}_{t-1}\frac{\beta\theta}{(1-\beta\theta)} + (1-\beta\theta)\,\hat{p}_t\frac{\beta\theta}{(1-\beta\theta)}$$

$$= (1-\beta\theta)\,\widehat{mc}_t + (1-\beta\theta)\left(\hat{p}_t - \hat{p}_{t-1}\right)$$

$$+\beta\theta(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\widehat{mc}_{t+k+1} + \hat{p}_{t+k+1} - \hat{p}_t\right\} + \beta\theta\left(\hat{p}_t - \hat{p}_{t-1}\right)$$

$$= (1-\beta\theta)\,\widehat{mc}_t + \pi_t + \beta\theta E_t\left\{\hat{p}^*_{t+1} - \hat{p}_t\right\}$$

$$\frac{\pi_t}{(1-\theta)} = (1-\beta\theta)\,\widehat{mc}_t + \pi_t + \beta\theta E_t \frac{\pi_{t+1}}{(1-\theta)}$$

$$\frac{1-(1-\theta)}{(1-\theta)}\pi_t = (1-\beta\theta)\,\widehat{mc}_t + \pi_t + \beta\theta E_t \frac{\pi_{t+1}}{(1-\theta)}$$

$$\pi_t = \frac{(1-\beta\theta)\,(1-\theta)}{\theta}\widehat{mc}_t + \pi_t + \beta E_t \pi_{t+1}$$

Implications

- *Exercise 9:* Show that (50) can be solved forward (assuming non-explosive behavior) to get

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k E_t\{\widehat{mc}_{t+k}\}. \tag{52}$$

- This means that inflation today can be expressed as a discounted sum of today's and future marginal costs
- This inflation dynamics is quite distinct to the one in the classical monetary economy
- **CLM**: an invisible hand moved inflation and other nominal variables such that they support an equilibrium allocation that was independent of the nominal variables themselves
- **NKM**: individual price setters are charging prices to maximize their profits. If they expect costs to rise, they anticipate that they might not be able to change prices in the future, and increase prices, and therefore inflation, already today

Bibliography I

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