A Small New Keynesian Model of the New Zealand Economy

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Abstract

This paper investigate whether a small open economy DSGE-based New Keynesian model can provide a reasonable description of key features of the New Zealand economy, in particular the transmission mechanism of monetary policy. The main objective is to design a simple, compact, and transparent tool for basic policy simulations. The structure of the model is largely motivated by recent developments in the area of DSGE modelling. Combining prior information and the historical data using Bayesian simulation techniques, we arrive at a set of parameters that largely reflect New Zealand's experience over the stable inflation targeting period. The resultant model can be used to simulate monetary policy paths and help analyze the robustness of policy conclusions to model uncertainty.

1 Introduction

One of the enduring research issues in central banks is the transmission of monetary policy. How does monetary policy affect key economic variables like output, inflation and the exchange rate – by what magnitude, and with how long and variable lags? As Lucas (1976) pointed out, traditional econometric models were unable to answer such questions, because their reduced-form parameters depended critically on the conduct of policy itself. As a result of the Lucas critique, central banks have focused on structural macro-economic models to guide policy. Since data samples are also often quite short, central banks have tended to calibrate the parameters of their structural models using a combination of economic theory and stylized macro-economic facts, rather than estimating them directly.

The macro-economic model used for forecasting and policy analysis at the Reserve Bank of New Zealand, the FPS model, is based on such calibration techniques. Although complemented by econometric information in recent years, FPS's parameters have not been estimated simultaneously. Among other things, this is due to the size and complexity of the model, rendering the model inappropriate for the use of simultaneous equation estimation techniques. The NZ Treasury Model (NZTM) is another large-scale macro model model used for policy purposes. Although the production block of NZTM is estimated using full information maximum likelihood, some elements, similar to FPS, are also calibrated rather than estimated.[1]

As the New Zealand economy has now had a stable macro-economic environment for more than a decade, it would seem desirable to find ways of utilizing the available historical data to complement and improve upon the Bank's existing policy models.

A number of smaller empirical models have also been used to investigate the characteristics of the New Zealand economy. Buckle, Kim and McLellan (2003) develop a structural vector autoregression and use it to investigate key drivers of New Zealand's business cycle. Though the paper takes an empirical approach to macroeconomic modelling, the weak theoretical foundation makes it vulnerable to the Lucas critique when using the model for policy simulations. Lees (2003) estimates a typical small open economy New Keynesian model, and uses the model to formulate optimal monetary pol-

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[1] Gardiner, Gray, Hargreaves and Szeto (2003) compare the dynamic properties of NZTM with FPS.

icy experiments for New Zealand. In the estimation of this latter model, agents' expectation behaviour is not explicitly taken into account nor does the estimation allow for the cross equation restrictions implied by the underlying structural parameters. There is therefore a niche available for a small empirical and theoretically-consistent model to complement existing monetary policy models in New Zealand. In this paper, we build on recent developments in the modelling literature to provide such a model.

Dynamic stochastic general equilibrium (DSGE) models with nominal rigidities, so-called 'New Keynesian' models, have become increasingly popular for the analysis of monetary policy. Examples include Bouakez, Cardia and Ruge-Murcia (2005), Christiano, Eichenbaum and Evans (2005) and references therein. In this paper, we investigate whether a DSGE-based small open economy model with nominal rigidities can provide a reasonable description of the New Zealand economy. In particular we are interested in the transmission of monetary policy and its response to shocks.

The design of our model builds extensively on previous work done in this area, notably by Smets and Wouters (2004), Gali and Monacelli (2005), Lubik and Schorfheide (2003), Monacelli (2005), Justiniano and Preston (2004), and Lubik and Schorfheide (2005). Among these studies, key aggregate relationships are derived from micro-foundations with optimizing agents and rational expectations. Models that are based on optimizing agents and deep parameters are less susceptible to the Lucas critique.

To confront the models with the data, some of these studies make use of Bayesian methods to combine prior judgements together with information contained in the historical data. The Bayesian approach also allows for the explicit evaluation of parameter and model uncertainty.

Bayesian DSGE modelling is a relatively new area of study. Consequently, such models have been infrequently applied to New Zealand. Previous studies using New Zealand data have also had a slightly different focus than our work here. Lubik and Schorfheide (2003) investigated whether exchange rate movements were an important factor in determining monetary policy for three small open economies, including New Zealand. A more recent study by Justiniano and Preston (2004) concentrated on the relative importance of nominal rigidities across different model specifications in explaining the data generating processes for three small open economies, including New Zealand.

The two studies just mentioned paid very little attention to the dynamic

behaviour of the New Zealand economy in response to various shocks and to the transmission of monetary policy – these two areas are the focus of this paper. We are also interested in whether or not the historical data is useful in recovering the deep parameters that underpin New Zealand's structural characteristics, using the Bayesian methodology.

The rest of the paper is structured as follows. Section 2 lays out the basic structure of our small open economy model. Section 3 summarizes the equilibrium structure of the model in log-linearized form. Section 4 briefly discusses the estimation methodology and the data used in estimating the model. In section 5, we first present the estimation results from the Bayesian simulations, then analyze the impact of various structural and non-structural innovations on the New Zealand economy. Finally, in section 6 we review our main findings and make suggestions for further work in developing the model.

2   A small open economy model

To make the paper self-contained, in this section we lay out the derivation of key structural equations implied by the model proposed by Gali and Monacelli (2005) and Monacelli (2005). The model's dynamics are enriched by allowing for external habit formation and indexation of prices, as in Justiniano and Preston (2004).

2.1   Households

The economy is inhabited by a representative household who seeks to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{U(C_t, H_t) - V(N_t)\} \tag{1}$$

$$U(C_t, H_t) = \frac{(C_t - H_t)^{1-\sigma}}{1 - \sigma} \quad \text{and} \quad V(N_t) = \frac{N_t^{1+\varphi}}{1 + \varphi}$$

where β is the rate of time preference, σ is the inverse elasticity of intertemporal substitution, and φ is the inverse elasticity of labour supply. N_t denotes hours of labour, and $H_t = hC_{t-1}$ represents external habit formation for the optimizing household, for $h \in (0, 1)$. C_t is a composite consumption index of

foreign and domestically produced goods defined as:

$$C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{2}$$

where $\alpha \in [0,1]$ is the import ratio measuring the degree of openness, and $\eta > 0$ is the elasticity of substitution between home and foreign goods. The aggregate consumption indices of foreign ($C_{F,t}$) and domestically ($C_{H,t}$) produced goods are given by:

$$C_{F,t} \equiv \left(\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

The elasticity of substitution between varieties of goods is assumed to be the same in the two countries ($\varepsilon > 0$).[2] The household's maximization problem is completed given the following budget constraint at time t:

$$\int_0^1 \{P_{H,t}(i)C_{H,t}(i) + P_{F,t}(i)C_{F,t}(i)\}di + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_t N_t \tag{4}$$

for $t = 1, 2, \ldots, \infty$, where $P_{H,t}(i)$ and $P_{F,t}(i)$ denote the prices of domestic and foreign good i respectively, $Q_{t,t+1}$ is the stochastic discount rate on nominal payoffs, D_t is the nominal payoff on a portfolio held at $t-1$ and W_t is the nominal wage.

Given the constant elasticity of substitution aggregator for $C_{F,t}$ and $C_{H,t}$ in equation (3), the optimal allocation for good i is given by the following demand functions:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \quad \text{and} \quad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \tag{5}$$

where $P_{H,t}$ is the price index of home produced goods, and $P_{F,t}$ is the import price index. Furthermore, assuming symmetry across all i goods, the optimal allocation of expenditure between domestic and imported goods is given by:

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \tag{6}$$

where $P_t \equiv \{(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\}^{\frac{1}{1-\eta}}$ is the overall consumer price index (CPI). Accordingly, total consumption expenditure for the domestic household is given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t$. Using this relationship, we can

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[2] The assumption is irrelevant given the small economy assumption. Domestic consumption of foreign goods should have a negligible influence on the foreign economy.

rewrite the intertemporal budget constraint in equation (4) as:

$$P_t C_t + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_t N_t \tag{7}$$

Solving the household's optimization problem yields the following set of first order conditions (FOCs):

$$(C_t - hC_{t-1})^{-\sigma}\frac{W_t}{P_t} = N_t^{\varphi} \tag{8}$$

$$\beta R_t E_t\left\{\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\right\} = 1 \tag{9}$$

where $R_t = 1/E_t Q_{t,t+1}$ is the gross nominal return on a riskless one-period bond maturing in $t+1$. The intra-temporal optimality condition in equation (8) states that the marginal utility of consumption is equal to the marginal value of labour at any one point of time; equation 9 gives the Euler equation for inter-temporal consumption. Log-linear approximations of equation (6) and the two FOCs yield:

$$c_{H,t} = -(1-\alpha)\{\eta(p_{H,t} - p_t) + c_t\} \tag{10}$$

$$c_{F,t} = -\alpha\{\eta(p_{F,t} - p_t) + c_t\} \tag{11}$$

$$w_t - p_t = \varphi n_t + \frac{\sigma}{1-h}\tilde{c}_t \tag{12}$$

$$\tilde{c}_t = E_t\tilde{c}_{t+1} - \frac{1-h}{\sigma}(r_t - E_t\pi_{t+1}) \tag{13}$$

where lower case letters denote the logs of the respective variables, $\tilde{c}_t = \frac{1}{1-h}(c_t - hc_{t-1})$, and $\pi_t = p_t - p_{t-1}$ is CPI inflation.

We assume households in the foreign economy face exactly the same optimization problem with identical preferences, and influence from the domestic economy is negligible. We arrive at a similar set of optimality conditions describing the dynamic behaviours of the foreign economy. However, assuming that the domestic economy is small relative to the foreign economy, foreign consumption approximately comprises only foreign-produced goods such that $C_t^* = C_{F,t}^*$ and $P_t^* = P_{F,t}^*$. Equations (12) and (13) continue to hold for the foreign economy with all variables taking a superscript (*).

Inflation, the real exchange rate and terms of trade

This section sets out some of the key relationships between inflation, the real exchange rate and the terms of trade. Throughout the paper, we maintain

the assumption that the *law of one price* (LOP) holds for the export sector, but incomplete pass-through for imports is allowed. The motivation behind this assumption is that New Zealand is a price taker with little bargaining power in international markets. For its export bundle, prices are determined exogenously in the rest of the world. On the import side, competition in the world market is assumed to bring import prices equal to marginal cost at the wholesale level, but rigidities arising from inefficient distribution networks and monopolistic retailers allow domestic import prices to deviate from the world price. Burstein, Neves and Rebelo (2003) provide a similar argument, which they support using United States (US) data. The mechanism of incomplete import pass-through will be formally discussed later.

We start by defining the *terms of trade* (TOT) as $S_t = \frac{P_{F,t}}{P_{H,t}}$ (or in logs $s_t = p_{F,t} - p_{H,t}$). The terms of trade is thus the price of foreign goods per unit of home good. Note, an increase in s_t is equivalent to an increase in competitiveness for the domestic economy because foreign prices increase and/or home prices fall. Log-linearizing the CPI formula around the steady state yields the following relationship between aggregate prices and the TOT:

$$
\begin{aligned}
p_t &\equiv (1 - \alpha)p_{H,t} + \alpha p_{F,t} \\
&= p_{H,t} + \alpha s_t
\end{aligned}
\tag{14}
$$

Taking the first difference of equation (14), we arrive at an identity linking CPI-inflation, domestic inflation ($\pi_{H,t}$) and the change in the TOT:

$$
\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{15}
$$

$$
\text{or} \quad \Delta s_t = \pi_{F,t} - \pi_{H,t} \tag{16}
$$

The difference between total and domestic inflation is proportional to the change in the TOT, and the coefficient of proportionality increases with the degree of openness, α. In addition, we define $\mathcal{E}_t$ as the nominal exchange rate (expressed in terms of foreign currency per unit of domestic currency). An increase in $\mathcal{E}_t$ coincides with an appreciation of the domestic currency. Similarly, we define the real exchange rate and the law of one price (LOP) gap as

$$
\zeta_t \equiv \frac{\mathcal{E}_t P_t}{P_t^*} \tag{17}
$$

and

$$
\Psi_t = \frac{P_t^*}{\mathcal{E}_t P_{F,t}} \tag{18}
$$

respectively. If LOP holds, ie if $\Psi_t = 1$, then the import price index $P_{F,t}$ is simply the foreign price index divided by $\mathcal{E}_t$, or $P_{F,t} = \frac{P_t^*}{\mathcal{E}_t}$. The LOP gap is a wedge or inverse mark-up between the *world* price of world goods and the *domestic* price of these imported world goods.

Substituting $\psi_t = \ln(\Psi_t)$ into the definition for s_t we get:

$$s_t = p_t^* - e_t - p_{H,t} - \psi_t \tag{19}$$

where e_t denotes the log of the nominal exchange rate, $\mathcal{E}_t$.

Next, we derive the relationship between s_t and the log real exchange rate $q_t = ln(\zeta_t)$. Substituting equation (19) into the definition of q_t, and using equation (14) gives:

$$
\begin{aligned}
q_t &= e_t + p_t - p_t^* \\
&= p_t - p_{H,t} - s_t - \psi_t \\
&= -\psi_t - (1 - \alpha)s_t
\end{aligned}
\tag{20}
$$

$$\Rightarrow \psi_t = -[q_t + (1 - \alpha)s_t]$$

Consequently, the LOP gap is inversely proportionate to the real exchange rate and the degree of international competitiveness for the domestic economy.

International risk sharing and uncovered interest parity

Under the assumption of complete international financial markets and perfect capital mobility, the expected nominal return from risk-free bonds, in domestic currency terms, must be the same as the expected domestic-currency return from foreign bonds, that is $E_t Q_{t,t+1} = E_t(Q_{t,t+1}^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t})$. Using this relationship, we can equate the intertemporal optimality conditions for the domestic and foreign households' optimization problem:

$$\beta E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \right\} = \beta E_t \left\{ \frac{P_t^*}{P_{t+1}^*} \frac{\mathcal{E}_t + 1}{\mathcal{E}_t} \left(\frac{\tilde{C}_{t+1}^*}{\tilde{C}_t^*} \right)^{-\sigma} \right\} \tag{21}$$

where $\tilde{C}_t = C_t - hC_{t-1}$ and $\tilde{C}_t^* = C_t^* - hC_{t-1}^*$. Assuming the same habit formation parameter across the two countries, the following relationship must hold in equilibrium:

$$C_t - hC_{t-1} = \vartheta(C_t^* - hC_{t-1}^*)\zeta_t^{-\frac{1}{\sigma}} \tag{22}$$

where ϑ is some constant depending on initial asset positions. Log-linearizing equation (22) around the steady gives:

$$
\begin{aligned}
c_t - hc_{t-1} &= (c_t^* - hc_{t-1}^*) - \frac{1-h}{\sigma}q_t \\
&= (y_t^* - hy_{t-1}^*) - \frac{1-h}{\sigma}q_t
\end{aligned}
\tag{23}
$$

The assumption of complete international financial markets recovers another important relationship, the *uncovered interest parity* condition:

$$
E_t\left(Q_{t,t+1}\{R_t - R_t^*\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\}\right) = 0
\tag{24}
$$

Log linearizing around the perfect foresight steady state yields the familiar UIP condition for the nominal exchange rate:[3]

$$
r_t - r_t^* = E_t\Delta e_{t+1}
\tag{25}
$$

Similarly, the real exchange rate can be expressed as:

$$
E_t\Delta q_{t+1} = -(r_t - \pi_{t+1}) - (r_t^* - \pi_{t+1}^*)
\tag{26}
$$

that is, the expected change in q_t depends on the current real interest rate differentials.

2.2   Firms

Production technology

There is a continuum of identical monopolistically-competitive firms; the jth firm produces a differentiated good, Y_j, using a linear technology production function:

$$
Y_t(j) = A_t N_t(j)
\tag{27}
$$

where $a_t \equiv \log A_t$ follows an AR(1) process, $a_t = \rho_a a_{t-1} + \nu_t^a$, describing the firm-specific productivity index. Aggregate output can be written as

$$
Y_t = \left[\int_0^1 Y_t(j)^{-(1-\varrho)}dj\right]^{-\frac{1}{1-\varrho}}.
\tag{28}
$$

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[3] The risk premium is assumed to be constant in the steady state.

Assuming a symmetric equilibrium across all j firms, the first order log-linear approximation of the aggregate production function can be written as:

$$y_t = a_t + n_t \tag{29}$$

Given the firm's technology, the real total cost of production is $TC_t = \frac{W_t}{P_{H,t}} \frac{Y_t}{A_t}$. Hence, the log of real marginal cost will be common across all domestic firms and given by:

$$mc_t = w_t - p_{H,t} - a_t \tag{30}$$

Price setting behaviour and incomplete pass-through

In the domestic economy, monopolistic firms are assumed to set prices in a Calvo-staggered fashion. In any period t, only $1 - \theta_H$, where $\theta_H \in [0,1]$, fraction of firms are able to reset its prices optimally, while the other fraction θ_H can not. Instead, the latter are assumed to adjust their prices, $P_t^I(j)$, by indexing it to last period's inflation as follows:

$$P_{H,t}^I(j) = P_{H,t-1}(j) \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\theta_H} \tag{31}$$

The degree of past inflation indexation is assume to be the same as the probability of resetting its prices.[4] We only consider the symmetric equilibrium case where $P_{H,t}(j) = P_{H,t}(k)$, $\forall j, k$. Let $\bar{P}_{H,t}$ denote the price level that optimizing firms set each period. Then the aggregate domestic price level will evolve according to:

$$P_{H,t} = \left\{ (1 - \theta_H)\bar{P}_{H,t}^{1-\varrho} + \theta_H \left[P_{H,t-1} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\theta_H} \right]^{1-\varrho} \right\}^{\frac{1}{1-\varrho}} \tag{32}$$

or in terms of inflation:

$$\pi_{H,t} = (1 - \theta_H)(\bar{p}_{H,t} - p_{H,t-1}) + \theta_H^2 \pi_{H,t-1} \tag{33}$$

When setting a new price, $\bar{P}_{H,t}$, in period t, an optimizing firm will seek to maximize the current value of its dividend stream subject to the sequence of demand constraints. In aggregate the following function is maximised:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} (\theta_H)^k E_t \left\{ Q_{t,t+k} \left(Y_{t+k}(\bar{P}_{H,t} - MC_{t+k}^n) \right) \right\} \tag{34}$$

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[4] The assumption ensures that the Phillips curve is vertical in the long run.

$$\text{subject to} \quad Y_{t+k} \leq \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} (C_{H,t+k} + C^*_{H,t+k})$$

where MC^n_{t+k} is the nominal marginal cost and the effective stochastic discount rate is now $\theta^k_H E_t Q_{t+k-1,t+k}$ to allow for the fact that firms have a $1-\theta_H$ probability of being able to reset prices in each period. The corresponding first order condition can be written as:[5]

$$\sum_{k=0}^{\infty} \theta^k_H E_t \left\{ Q_{t,t+k} Y_{t+k} (\bar{P}_{H,t} - \frac{\varepsilon}{1-\varepsilon} MC^n_{t+k}) \right\} = 0 \qquad (35)$$

where $\frac{\varepsilon}{1-\varepsilon}$ is the real marginal cost if prices were fully flexible. Substituting out $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$ from the consumption Euler equation in (9) yields:

$$\sum_{k=0}^{\infty} (\beta\theta_H)^k P_t^{-1} C_t^{-\sigma} E_t \left\{ P_{t+k}^{-1} C_{t+k}^{-\sigma} Y_{t+k} (\bar{P}_{H,t} - \frac{\varepsilon}{1-\varepsilon} MC^n_{t+k}) \right\} = 0 \qquad (36)$$

Since $P_t^{-1} C_t^{-\sigma}$ is known at date t, it can be taken out of the expectation summation, after rearranging yields:

$$\sum_{k=0}^{\infty} (\beta\theta_H)^k E_t \left\{ P_{t+k}^{-1} C_{t+k}^{-\sigma} Y_{t+k} (\bar{P}_{H,t} - \frac{\varepsilon}{1-\varepsilon} MC^n_{t+k}) \right\} = 0$$

$$(37a)$$

$$\sum_{k=0}^{\infty} (\beta\theta_H)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{H,t-1}}{P_{t+k}} \left(\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon-1} MC_{t+k} \frac{P_{H,t+k}}{P_{H,t-1}} \right) \right\} = 0$$

$$(37b)$$

where $MC_{t+k} = \frac{MC^n_{t+k}}{P_{H,t+k}}$ is the real marginal cost. Log-linearizing equation (37b) around the steady state to obtain the decision rule for $\bar{p}_{H,t}$ gives:

$$\bar{p}_{H,t} = p_{H,t-1} + \sum_{k=0}^{\infty} (\beta\theta_H)^k \{E_t \pi_{H,t+k} + (1 - \beta\theta_H) E_t mc_{t+k}\} \qquad (38)$$

that is, firms set their prices according to the future discounted sum of inflation and deviations of real marginal cost from its steady state. We can

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[5] See the appendix in Gali and Monacelli (2005).

rewrite equation (38) as:

$$\begin{aligned}
\bar{p}_{H,t} &= p_{H,t-1} + \pi_{H,t} + (1 - \beta\theta_H)mc_t \\
&\quad + (\beta\theta_H)\sum_{k=0}^{\infty}(\beta\theta_H)^k\{E_t\pi_{H,t+k+1} + (1 - \beta\theta_H)E_t mc_{t+k+1}\} \\
&= p_{H,t-1} + \pi_{H,t} + (1 - \beta\theta_H)mc_t + \beta\theta_H(\bar{p}_{H,t+1} - p_{H,t}) \\
\bar{p}_{H,t} - p_{H,t-1} &= \beta\theta_H E_t\pi_{H,t+1} + \pi_{H,t} + (1 - \beta\theta_H)mc_t
\end{aligned} \tag{39}$$

The first line involves splitting up the summation into two terms, one at date t and other from $t+1$ to ∞; the second line rewrites the last term using equation (38); lastly, rearrange to obtain the familiar NKPC equation. Substituting equation (39) back into (33), and then rearranging, we obtain the evolution of domestic inflation as:

$$\pi_{H,t} = \beta(1 - \theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \lambda_H mc_t \tag{40}$$

where $\lambda_H = \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H}$. The Calvo pricing structure yields a familiar New Keynesian Phillips Curve (NKPC), that is, the domestic inflation dynamic has both a forward looking component and a backward-looking component. If all firms were able to adjust their prices at each and every period, i.e: $\theta_H = 0$, the inflation process would be purely forward looking and disinflationary policy would be completely costless. The real marginal costs faced by the firm are also an important determinant of domestic inflation.

Here we assume the LOP holds at the wholesale level for imports. However, inefficiency in distribution channels together with monopolistic retailers keep domestic import prices over and above the marginal cost. As a result, the LOP fails to hold at the retail level for domestic imports. Following a similar Calvo-pricing argument as before, the price setting behaviour for the domestic importer retailers could be summarized as[6]:

$$\bar{p}_{F,t} = p_{F,t-1} + \sum_{k=0}^{\infty}(\beta\theta_F)^k\{E_t\pi_{F,t+k} + (1 - \beta\theta_F)E_t\psi_{t+k}\} \tag{41}$$

where $\theta_F \in [0,1]$ is the fraction of importer retailers that cannot re-optimize their prices every period. In setting the new price for imports, domestic retailers are concerned with the future path of import inflation as well as the LOP gap, ψ_t. Essentially, ψ_t is the margin over and above the wholesale import price. A non-zero LOP gap represents a wedge between the *world* and

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[6] See Gali and Monacelli (2005) for more detail.

domestic import prices. This provides a mechanism for incomplete import pass-through in the short-run, implying that changes in the world import prices have a gradual affect on the domestic economy. Substituting equation (41) into the determination of $\pi_{F,t}$ arising from the Calvo-pricing structure yields:

$$\pi_{F,t} = \beta(1 - \theta_F)E_t\pi_{F,t+1} + \theta_F\pi_{F,t-1} + \lambda_F\psi_t \tag{42}$$

where $\lambda_F = \frac{(1-\beta\theta_F)(1-\theta_F)}{\theta_F}$. Log-linearizing the definition of CPI and taking the first difference yields the following relationship for overall inflation:

$$\pi_t = (1 - \alpha)\pi_H + \alpha\pi_F \tag{43}$$

Taking the definition for overall inflation in (43) together with equations (40) and (42) completes the specification of inflation dynamics for the small open economy.

In general, inflation dynamics in sticky-price models are mainly driven by firms' preference for smoothing their pricing decisions. This gives rise to nominal rigidities we would not otherwise see if prices were fully flexible. The *cost* of inflation in this case is essentially the cost to the economy arising from prices not being able to adjust, hence the classification of such models in the literature as 'New Keynesian'. Using the cost of adjustment argument for the firm's pricing decisions yields a similar NKPC relationship as shown in Yun (1996). From the social planner's perspective, optimal policy is one that minimizes deviations of marginal cost and the LOP gap from its steady state. Here we do not set out an explicit optimization program for the social planner, instead, we assume that the central bank follows a simple reaction function to try to replicate the fully flexible price equilibrium.

3 Equilibrium

3.1 Aggregate demand and output

Goods market clearing in the domestic economy requires that domestic output is equal to the sum of domestic consumption and foreign consumption of home produced goods (exports):

$$y_t = (1 - \alpha)c_{H,t} + \alpha c_{H,t}^* \tag{44}$$

Acknowledging that

$$C_{H,t} = (1 - \alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \tag{45}$$

and

$$C_{H,t}^* = \alpha \left(\frac{\varepsilon_t P_{H,t}}{P_t^*} \right)^{-\eta} C_t^*, \tag{46}$$

log linearizing the two demand functions gives:

$$
\begin{aligned}
c_{H,t} &= -\eta(p_{H,t} - p_t) + c_t \\
&= \alpha\eta s_t + c_t
\end{aligned}
\tag{47}
$$

$$
\begin{aligned}
c_{H,t}^* &= -\eta(e_t + p_{H,t} - p_t^*) + c_t^* \\
&= -\eta(p_{H,t} - p_{F,t} - \psi_t) + c_t^* \\
&= \eta(s_t + \psi_t) + c_t^*
\end{aligned}
\tag{48}
$$

From equation (47), an increase in s_t (equivalent to an increase in domestic competitiveness in the world market) will see domestic agents substitute out of foreign-produced goods into home-produced goods for a given level of consumption. The magnitude of substitution will depend on η, the elasticity of substitution between foreign and domestic goods; and the degree of openness, α. Similarly, from equation (48) an increase in s_t will see foreigners substitute out of foreign goods and consume more home goods for a given level of income.

Substituting equations (47) and (48) into (44) yields the goods market clearing condition for the small open economy:

$$
\begin{aligned}
y_t &= (1 - \alpha)[\eta\alpha s_t + c_t] + \alpha[\eta(s_t + \psi_t) + c_t^*] \\
&= (1 - \alpha)c_t + \alpha c_t^* + (2 - \alpha)\alpha\eta s_t + \alpha\eta\psi_t
\end{aligned}
\tag{49}
$$

Notice that when $\alpha = 0$, the closed economy case, we have $y_t = c_t$.

3.2   Marginal cost and inflation dynamics

In section 2.2, we derived the evolution of domestic inflation arising from Calvo-style pricing behaviour as:

$$\pi_{H,t} = \beta(1 - \theta)E_t\pi_{H,t+1} + \theta\pi_{H,t-1} + \lambda_H mc_t \tag{50}$$

where $\lambda_H = \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H}$. From equation (30), the real marginal cost faced by the monopolistic firm (assuming a symmetric equilibrium) is:

$$
\begin{aligned}
mc_t &= w_t - p_{H,t} - a_t \\
&= (w_t - p_t) + (p_t - p_{H,t}) - a_t \\
&= \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \varphi n_t + \alpha s_t - a_t \\
&= \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \varphi y_t + \alpha s_t - (1+\varphi)a_t
\end{aligned}
\tag{51}
$$

The third equality uses the FOC in equation (12), whereas the fourth one rewrites n_t using the linearized production function in equation (29). Thus, we see that the marginal cost is an increasing function of domestic output and s_t, and is inversely related to the level of labour productivity.

3.3   A simple reaction function

To complete the small open economy model, we need to specify the behaviour of the domestic monetary authority. Optimal policy in this particular model is one which replicates the fully flexible price equilibrium such that $\pi_t = y_t - \bar{y}_t = 0$ as discussed earlier. The aim of the monetary authority is to stabilize both output and inflation to try to reproduce this equilibrium. Rather than setting out an explicit optimizing program for the monetary authority, as in Woodford (2003) and Clarida, Gali and Gertler (2001), we assume the monetary authority follows a simple reaction function. Optimal policy under sticky-price settings is approximated using the following reaction function:

$$
r_t = \rho_r r_{t-1} + (1 - \rho_r)[\phi_1 \pi_t + \phi_2 \Delta y_t]
\tag{52}
$$

where ρ_r is the degree of interest rate smoothing, ϕ_1 and ϕ_2 are the relative weights on inflation and output growth respectively. Here we are estimating the model using a *speed limit* policy rather than the traditional Taylor rule based on the output gap and inflation.

3.4   The linearized model

The foreign sector is assumed to be exogenous to the small open economy. Furthermore, the behaviour of the foreign sector is summarized by a system of two-equations in output and the real interest rate. Appendix A provides

a summary of the linearized model consisting of 11 equations for the endogenous variables, and 3 equations for the exogenous processes.

The log-linearized model can be written as a linear rational expectations (LRE) system in the form of:

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dz_t \tag{53}$$
$$0 = E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right] \tag{54}$$
$$z_{t+1} = Nz_t + \nu_{t+1}; \quad E_t[\nu_{t+1}] = 0 \tag{55}$$

where

$$x_t = \left\{y_t, q_t, r_t, \pi_t, \pi_{F,t}, r_t^*, y_t^*\right\}$$

is the endogenous state vector,

$$y_t = \left\{\psi_t, s_t, c_t, mc_t, \pi_{H,t}\right\}$$

is the other endogenous vector, and

$$z_t = \left\{a_t, \nu_t^s, \nu_t^q, \nu_t^{\pi_H}, \nu_t^{\pi_F}, \nu_t^r, \nu_t^{y^*}, \nu_t^{r^*}\right\}$$

is a vector of exogenous stochastic processes underlying the system. Matrix A and B are of size 3×7, C is 3×5, D is 3×8, F, G and H are 10×7, J and K are 10×5, and L and N are 10×8. Solving the system of equations from (53) to (55), using the algorithm from Uhlig (1995), yields the following recursive equilibrium law of motion:

$$x_t = Px_{t-1} + Qz_t \tag{56}$$
$$y_t = Ry_{t-1} + Sz_t \tag{57}$$

such that the equilibrium described by the matrices P, Q, R and S is stable.

4   Empirical analysis

This section outlines the procedure used to obtain the posterior distribution of the structural parameters underlying the model described in sections 2 and 3.

4.1 The Bayesian approach

In recent years, substantial improvements in computational technology has seen the use of Bayesian methods populate throughout the economics literature, especially in open economy DSGE modelling. Recent examples include Smets and Wouters (2003), Justiniano and Preston (2004), and Lubik and Schorfheide (2005). The Bayesian approach facilitates comparison between non-nested models and allows the user to treat model and parameter uncertainty explicitly. Bayesian modellers recognize that "all models are false", rather than assuming they are working with the correct model. This perspective contrasts with classical methods that search for the single model with the highest posterior probability given the *evidence*. Bayesian inference is in terms of probabilistic statements about unknown parameters rather than classical hypothesis testing procedures associated with notional repeated samples.

In the Bayesian context, all information about the parameter vector θ is contained in the posterior distribution. All information about θ from the *data* is conveyed through the likelihood: the likelihood principle always holds. For a particular model i, the posterior density of the model parameter θ can be written as:

$$p(\theta|Y^T, i) = \frac{L(Y^T|\theta, i)p(\theta|i)}{\int L(Y^T|\theta, i)p(\theta|i)d\theta} \tag{58}$$

where $p(\theta|i)$ is the prior density and $L(Y^T|\theta, i)$ is the likelihood conditional on the observed data Y^T. An important part of the Bayesian approach is to find a model i that maximizes the posterior probability given by $p(\theta|Y^T, i)$.

The likelihood function can be computed via the state-space representation of the model together with the measurement equation linking the observed data and the state vector. The economic model described in sections 2 and 3 has the following (approximate) state-space representation:

$$S_{t+1} = \Gamma_1 S_t + \Gamma_2 w_{t+1} \tag{59}$$
$$Y_t = \Lambda S_t + \mu_t \tag{60}$$

where $S_t = \{x_t, y_t\}$ from equations (56) and (57), w_t is a vector of state innovations, Y_t is a $k \times 1$ vector of observed variables, and μ_t is regarded as measurement error. The matrices Γ_1 and Γ_2 are functions of the model's deep parameters (or P, Q, R and S), and Λ defines the relationship between the observed and state variables. Assuming the state innovations and measurement errors are normally distributed with mean zero and variance-covariance

matrices Ξ and Υ respectively, the likelihood function of the model is given by:

$$
\begin{aligned}
\ln L(Y^T|\Gamma_1, \Gamma_2, \Lambda, \Xi, \Upsilon) &= \frac{TN}{2} \ln 2\pi \\
&+ \sum_{t-1}^{T} \left[\frac{1}{2} \ln |\Omega_{t|t-1}| + \frac{1}{2} \mu'_t \Omega_{t|t-1}^{-1} \mu_t \right] \quad (61)
\end{aligned}
$$

Recognizing that $\int L(Y^T|\theta, i) p(\theta|i) d\theta$ is constant for a particular model i, we only need to be able to evaluate the posterior density up to a proportionate constant using the following relationship:

$$
p(\theta|Y^T) \propto L(Y^T|\theta) p(\theta) \quad (62)
$$

The posterior density can be seen as a way of summarizing information contained in the likelihood weighted by the prior density $p(\theta)$. The prior can bring to bear information that is not contained in the sample, Y^T. Given the sequence of $\{\theta^j\}_1^N \sim p(\theta|Y^T)$, by the law of large numbers:

$$
E_\theta[g(\theta)|Y^T] = \frac{1}{N} \sum_{j=1}^{N} g(\theta^j) \quad (63)
$$

where $g(.)$ is some function of interest. The sequence of posterior draws $\{\theta^j\}_1^N$ used in evaluating equation (63) can be obtained using Markov chain Monte Carlo (MCMC) methods. We use the random walk Metropolis Hastings algorithm as described in Lubik and Schorfheide (2005) to generate the Markov chains (MC) for the model's parameters.[7]

4.2   Data and priors

Data from 1991Q1 to 2004Q4 for New Zealand is used in the analysis of our small open economy New Keynesian model. Quarterly observations on domestic output per capita (y_t), interest rates (r_t), overall inflation (π_t), import inflation ($\pi_{F,t}$), real exchange rate (q_t), the competitive price index or equivalently terms of trade (s_t), foreign output (y_t^*) and real interest rate ($\bar{r}_t^* = r_t^* - \pi_t^*$) are taken from Statistics New Zealand and the Reserve Bank of New Zealand. All variables are re-scaled to have a mean of zero and could

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[7] We also used DYNARE for preliminary investigation of the model.

be interpreted as an approximate percentage deviation from the mean.[8] See Appendix B for a more detailed description of the data transformations.

The choice of priors for our estimation are guided by several considerations. At a basic level, the priors reflect our beliefs and the confidence we have about the likely location of the structural parameters. Information on the structural characteristics of the New Zealand economy, such as the degree of openness, being a commodity producer and its institutional settings, were all taken into account. In the case of New Zealand, micro-level studies were relatively scarce. Priors from similar studies using New Zealand data for example Lubik and Schorfheide (2003), and Justiniano and Preston (2004) were also considered. The Reserve Bank's main macro model FPS is taken as a good approximation of the Bank's view of the New Zealand economy, and key parameters contained in FPS and its implied dynamic properties were used to inform our choice of priors. Finally, the choice of prior distributions reflect restrictions on the parameters such as non-negativity or interval restrictions. Beta distributions were chosen for parameters that are constrained on the unit-interval. Gamma and normal distributions were selected for parameters in $\Re^+$, while the inverse gamma distribution was used for the precision of the shocks.

The priors on the model's parameters are assumed to be independent of each other, which allows for easier construction of the joint prior density used in the MCMC algorithm. Furthermore, the parameter space is truncated to avoid indeterminacy or non-uniqueness in the model's solution. The marginal prior distributions for the model's parameters are summarized in table (1).

4.3   Estimation and convergence diagnostics

To avoid the problem of stochastic singularity in the case where there are more observed variables than the number of shocks, three *iid* N(0,1) measurement errors were added to the LRE system. An advantage of adding measure errors is that it allows one to change the observed variables for the model across different specifications. ν_t^s and ν_t^q can be interpreted as deviations from the definition of the TOT and UIP implied by the model. Lastly, ν_t^r can be used to measure the monetary surprises by the Reserve Bank as deviations from the specified reaction function.

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[8] Apart from the interest rate and inflation data which are already in percentage terms.

Given the data and the prior specifications in section 4.2, we generate two parallel 1,500,000 draws[9] of the Markov chain using the method described earlier. The Markov chain is generated conditional on the degree of openness (α) and the time preference (β) parameters, which are fixed at 0.4 and 0.99 respectively.

Various convergence diagnostic statistics were computed. The first column of Table (2) shows the mean of the posterior distribution. The NSE refers to the numeric standard error as the approximation of the true posterior moment and the p-value is the test between the means generated from two independent chains as in Geweke (1999). There is no indication that the two means are significantly different from each other. The fourth column shows the univariate "shrink factor" using the ratio of between and within variances as in Brooks and Gelman (1998). A shrink factor close to 1 is evidence for convergence to a stationary distribution. The autocorrelations statistics are shown in column 5 to 8. The multivariate "shrink factor" was computed to be 1.0117. All MCMC diagnostic tests suggest that the Markov chains has converged to its stationary distribution after 1.5 million iterations.

5 Estimation results

5.1 Posterior parameter estimates

Based on the two independent Markov Chains,[10] we compute the posterior mean, median and the 95 percent probability intervals for each of the parameters, with results reported in Table 3. The prior and the estimated posterior marginal densities are plotted in Figure 1. The plots indicate there is a significant amount of information contained in the data that can be used to update our prior beliefs about the model's parameters. The posterior marginal densities are much more concentrated compare than the prior densities with the exception of ϕ (the inverse elasticity of labour supply).

Our results show there is a relatively high degree of external habit persistence with $h^m = 0.92$ (the superscript m denotes the median of the posterior

---

[9] Each chain is generated at different starting values. It takes approximately 120 CPU hours to generate each independent chain using the APAC linux cluster machine.

[10] The reported statistics are computed after eliminating the first 40 percent of the Markov Chain (burn-in).

distribution) compared with other studies for the US and the euro area, eg Smets and Wouters (2004) and Lubik and Schorfheide (2005). The median of the inverse elasticity of intertemporal substitution, σ, is estimated to be 0.39. Smaller values of σ imply that households are less willing to accept deviations from a uniform pattern of consumption over time. This low value seems to be consistent with the relatively high degree of habit persistence discussed earlier. The posterior median for the elasticity of substitution between home and foreign goods, η, is around 0.85. The relatively low value for η is in line with our prior that New Zealand is a commodity-producer and its consumption basket relies heavily on foreign produced goods. The estimated inverse elasticity of substitution for labour, φ, turns out to be much greater than 1. This means a 1 percent increase in the real wage will result in only a small change in labour supply.

On the supply side, the median estimate of the probability of not changing price in a given quarter, or equivalently the proportion of firms that do not re-optimize their prices in a given quarter, is around 75 percent for domestic firms and slightly lower for import retailers at 72 percent. These Calvo coefficients imply that the average duration of price contracts is around four quarters for domestic firms and three quarters for import retailers.[11] This aggregate degree of nominal price rigidity is much lower than that reported for the euro area, but is comparable with estimates for the US.

The simple reaction function used in the model provides a fairly good description of monetary policy over the stable inflation period in New Zealand. The posterior median for the degree of interest rate smoothing is estimated to be 0.72 with 1.45 and 0.41 being the weight on inflation and output respectively.

5.2 Impulse response analysis

Figures 2 to 7 plots the impulse functions of the economy in respond to a one unit increase in the various structural and non-structural shocks. Due to computational difficulties, these impulses were calculated using 10,000 random draws from the model's empirical posterior distribution rather than taking the full 1.5 million Markov chains. The median impulse responses are drawn in solid lines while the dotted lines represent the 5th and 95th percentiles evaluated at each point in time.

---

[11] Duration $= \frac{1}{1-\theta_i}$.

Figure 2 shows that following a temporary positive labour productivity shock, consumption is higher on impact and stays above zero until some 30 quarters later. Output, on the other hand, stays relatively static on impact and decrease slightly before staying above trend until 40 quarters later. The strong persistence in the impulse response can be attributed to the high degree of persistence in the estimated productivity shock.[12] The initial decrease in output suggests that agent's substitution between working and leisure dominates the lower cost of production that arises from the increase in productivity. The impulse analysis suggests that the positive impact on output does not come around until five to six quarters later. Inflation falls initially as the higher labour productivity helps reduce the cost of production before returning close to zero 40 quarters later. In this particular case, the monetary authority can afford to loosen monetary policy to bring inflation back to zero. The exchange rate initially appreciates before responding to the monetary loosening via the UIP condition.

Figure 3 shows the effects of a positive import inflation shock. Both domestic and overall inflation are higher on impact, with higher import prices pushing up the cost of production. The higher foreign prices relative to domestic prices increase the degree of competitiveness for the domestic economy. This will see domestic agents substitute out of foreign produced goods into home produced goods in response to the price signal – the expenditure switching effect from improvements in the domestic economy's terms of trade. From the impulse responses, this has a positive and significant impact on domestic output. The monetary authority responds to the higher overall inflation and output by raising interest rates by 40 basis point before slowly returning to equilibrium over 8 quarters. The higher interest rate appreciates the exchange rate which acts as another channel to bring both import and domestic inflation back to zero.

Figure 4 shows the effects of a positive domestic inflation shock (which can be interpreted as a supply shock). Initially, the higher rate of domestic inflation relative to import inflation decreases the degree of domestic competitiveness by about 1 percent. The monetary authority respond to the higher rate of inflation by increasing the interest rate by 28 basis points and reaching a peak of 38 basis points in the third quarter before slowly coming back to equilibrium. Domestic output stays relatively static initially before decreasing

---

[12] The estimated AR(1) coefficient of 0.98 on the labor productivity tend to suggest a unit root maybe present in the linearly detrended output data.

in response to the monetary tightening. There a significant output-inflation tradeoff in face of the aggregate supply shock. The monetary tightening also leads to an appreciation of the exchange rate which act as another channel to bring inflation back to equilibrium.

Figure 5 shows a monetary tightening, a 1 percent increase in the interest rate. Immediately after the shock, consumption, output and overall inflation fall by 0.6 percent, 0.8 percent and 0.4 percent respectively. The consumption response is hump-shaped because, under habit formation, agents smooth both the level and the change of consumption. The peak consumption response takes place after two quarters. Output, on the other hand, does not experience the same hump-shape response. The inclusion of stock adjustment, in particular introducing capital into the model, may help improve the dynamic response of output, enabling it to reproduce the hump-shaped response usually found in VAR models.

The 1 percent increase in the interest rate has a peak influence on overall inflation of 0.5 percent, which is reached after three quarters. Due to the initial negative impact on output and inflation, the 1 percent interest rate innovation only results in a 0.6 percent increase in the nominal interest rate. This decreases quickly below zero after 2 quarters before adjusting slowly to restore both inflation and output back to its equilibrium. The exchange rate reacts positively to the monetary tightening before returning to equilibrium. The model predicts that a 1 percent interest rate shock will result in close to a 2 percent appreciation of the exchange rate.

Figure 6 shows a 1 percent deviation of the exchange rate from the UIP condition. Since no extrinsic persistence is assumed for the process of this shock, the shock has a relatively short-lived effect on the real exchange rate. Consequently, output will be 0.2 percent lower while there will be little effect on consumption. On the other hand, the higher exchange rate decreases both import and domestic inflation with overall inflation falling by 0.1 percent. The UIP shock also results in a small monetary expansion: interest rates decline by 10 basis points. The temporary exchange rate appreciation has little influence on the domestic economy's terms of trade.

Figure 7 shows a 1 percent increase in s_t, which corresponds to an improvement in international competitiveness for the domestic economy. Following the shock, there is an increase in aggregate demand that causes output to increase while leaving inflation pretty much unchanged. There is no change in overall inflation, due to the opposing effects from import and domestic

inflation. This shock has a very minor effect on domestic consumption. One can think of the differentials in the output and consumption paths as being attributed to the increase in exports. The interest rate response is hump-shaped; the peak response (8 basis points) takes place after three quarters. There is a small exchange rate appreciation in response to the monetary tightening which helps offset the higher TOT to some degree. It is not surprising that the 1 percent shock has a very minor quantitative impact on the New Zealand economy given that the estimated standard deviation on terms of trade shock is around 9 percent.

6 Concluding remarks

In this paper, we used Bayesian methods to combine prior information with the historical data, to develop a small open economy model of the New Zealand economy. The Bayesian approach provided us with a tool to understand and learn about the sources of uncertainty imbedded in quantitative macroeconomic modelling. The model and the set of estimated parameters can be used to guide future monetary policy questions in New Zealand.

The estimated parameters largely reflected New Zealand's structural characteristics of being a small, relatively open commodity producer. Here we summarize our main empirical findings. First, there is a high degree of habit formation together with a relatively low degree of intertemporal consumption substitutability. Agents are quite reluctant to deviate from a uniform consumption pattern over time. In response to shocks, consumption deviations from equilibrium appears to be relatively small when compared with output variation. When adjustments are necessary, there is a high degree of smoothing to both the level and the change of consumption. Second, the low degree of substitutability between home and foreign produced goods reflects New Zealand's commodity-producer characteristic; New Zealand's consumption basket relies heavily on foreign produced goods that are not close substitutes for the commodities New Zealand specialises in producing. Third, the low elasticity for labour supply decisions is partly attributed to the relatively immobile work force. However, explicit modelling of the labour market may be required to gain further insights. Fourth, the average duration of price contracts was estimated to be around five quarters for domestic firms and four quarters for import retailers. Lastly, the impulse response functions presented in this paper provide a qualitative and quantitative way of under-

standing the dynamic behaviour of the economy in response to the various shocks. These impulse response also enable us to describe the transmission of monetary policy to the rest of the economy.

In this paper, we restricted ourselves to a relatively simple specification of the model with only two sources of nominal rigidities, a linear production function in labour, and a simple role for the central bank. In future research, it would be interesting to expand the model to incorporate other factors of interest to policymakers, including: (i) capital accumulation and investment rigidities; (ii) an explicit government sector with a role for fiscal policy and interactions with monetary policy; (iii) labour market rigidities; (iv) in a multi sector/good setting; (v) an endogenous foreign sector; and (vi) incomplete financial markets, to faciliate analysis of current account dynamics. Lastly, it would also be interesting to investigate the forecasting ability of this DSGE structure.

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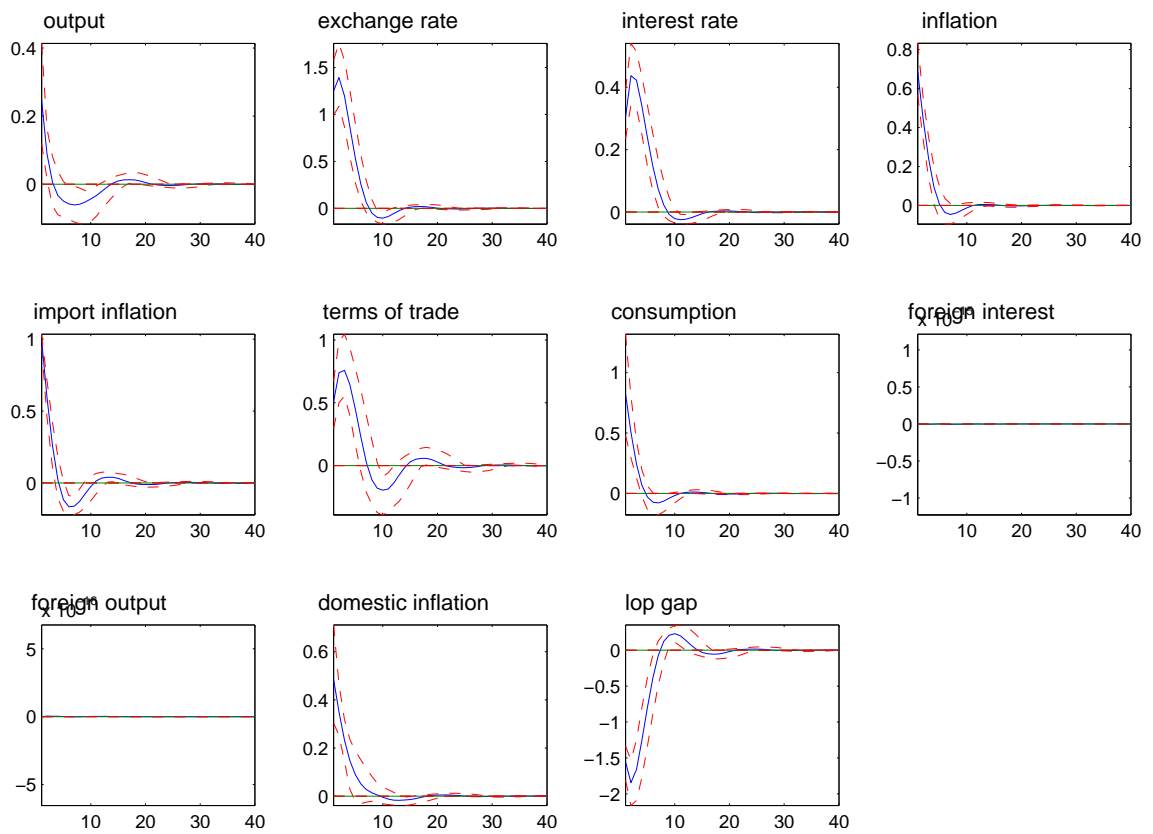
Figure 1
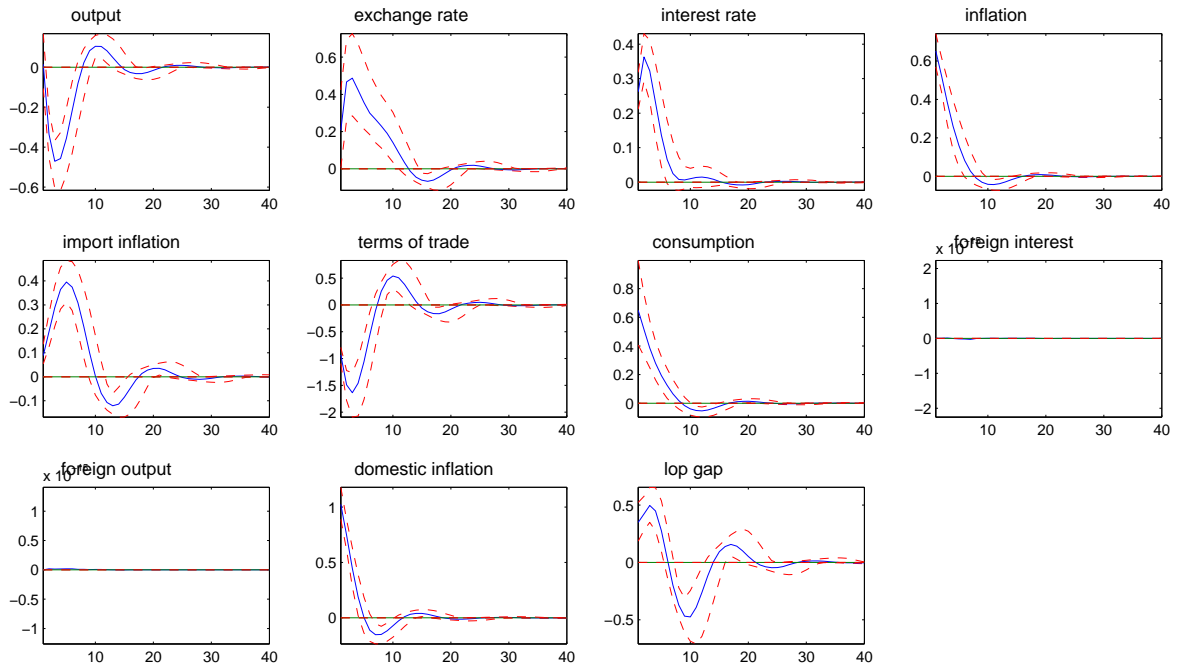**Posterior and prior marginal density plot**
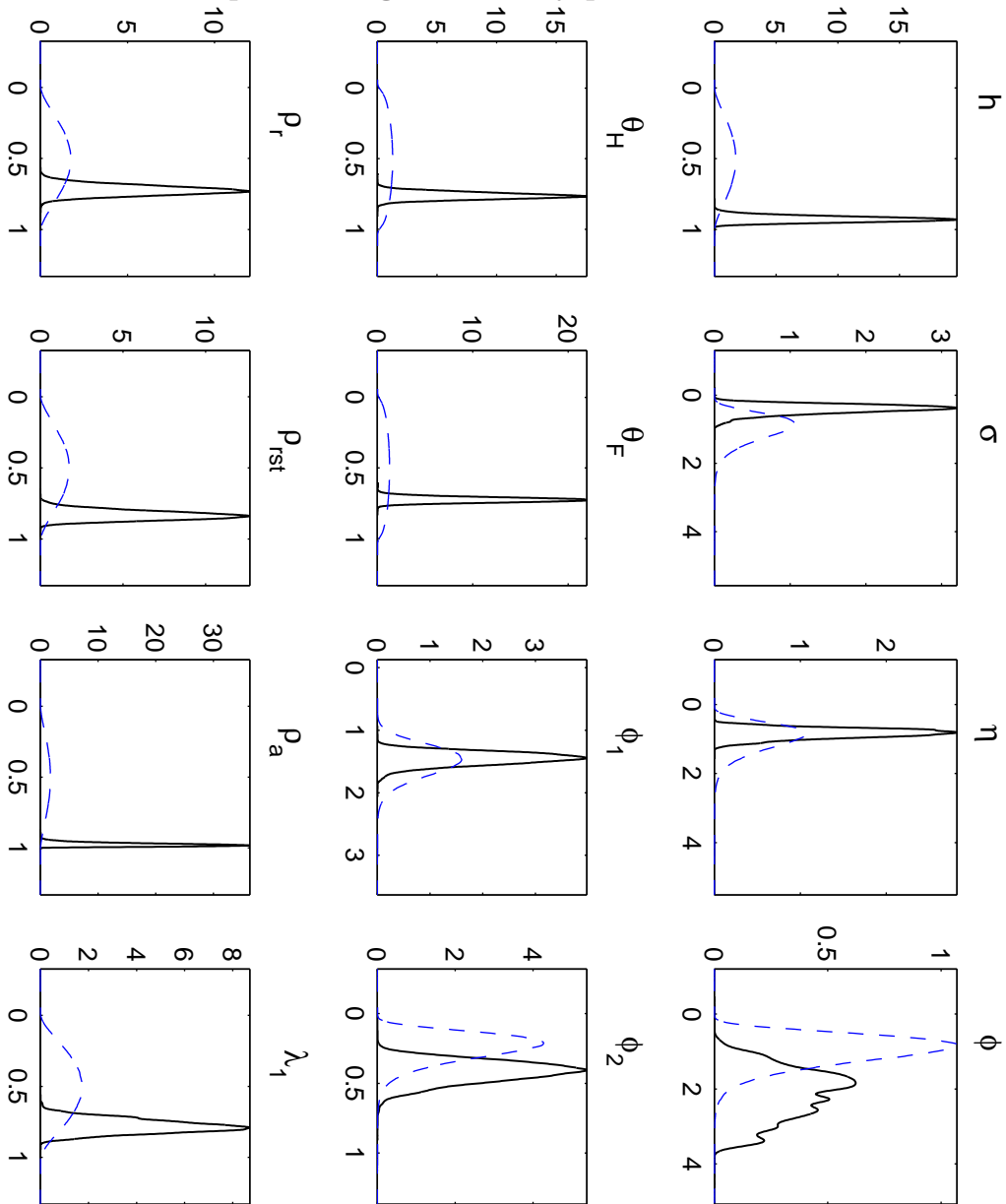
Figure 2
Impulse response functions from one unit of labour productivity innovation.
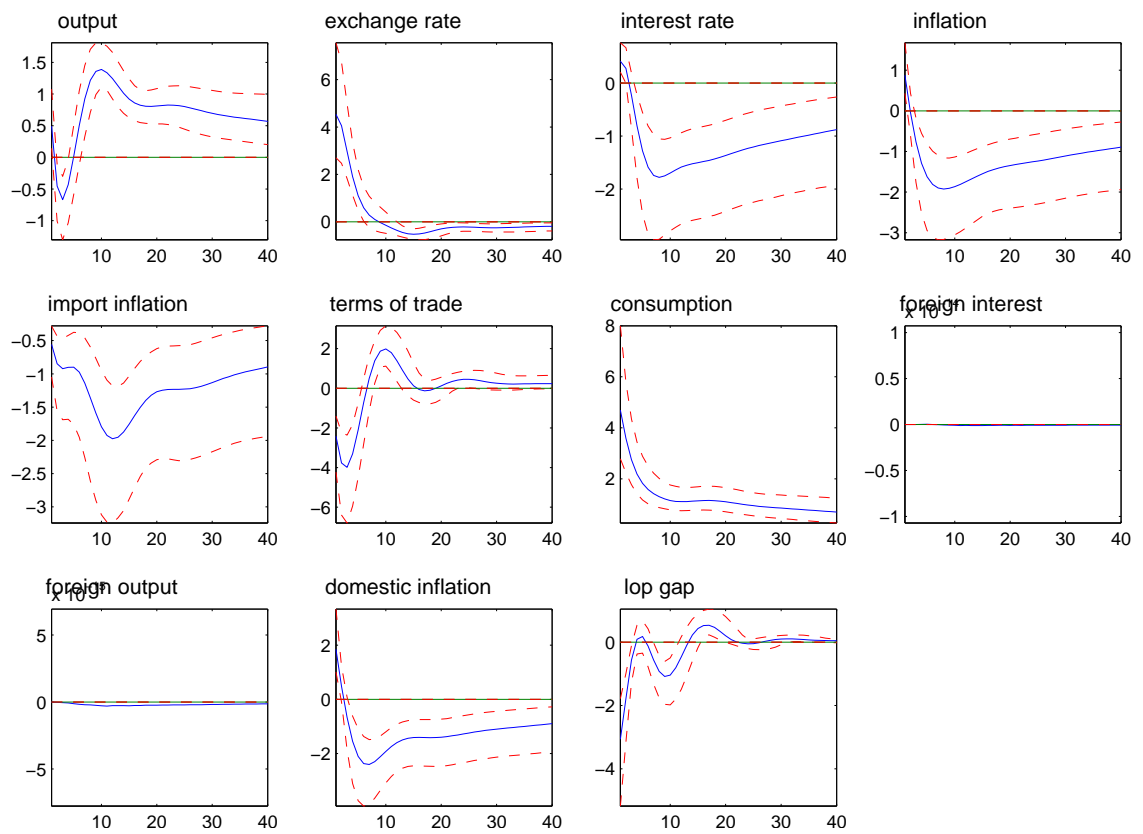
Figure 3

Impulse response functions from one unit of import inflation innovation.
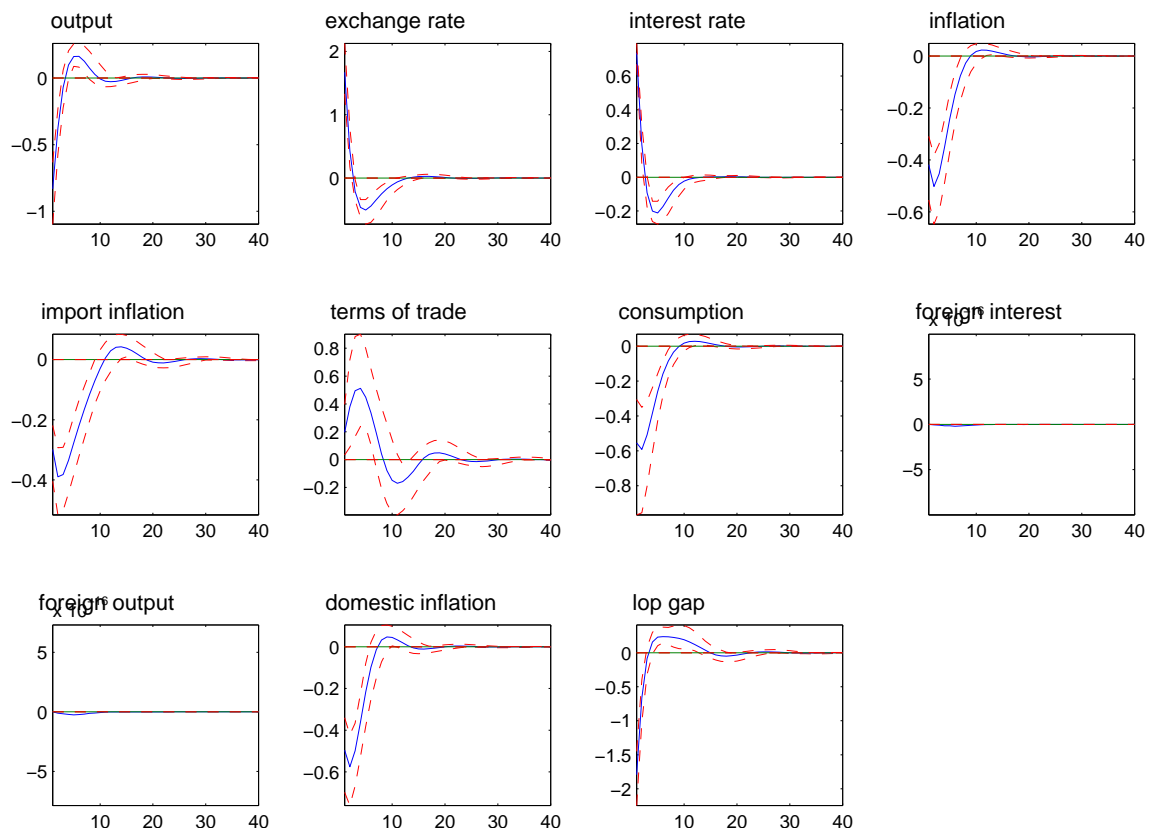
Figure 4
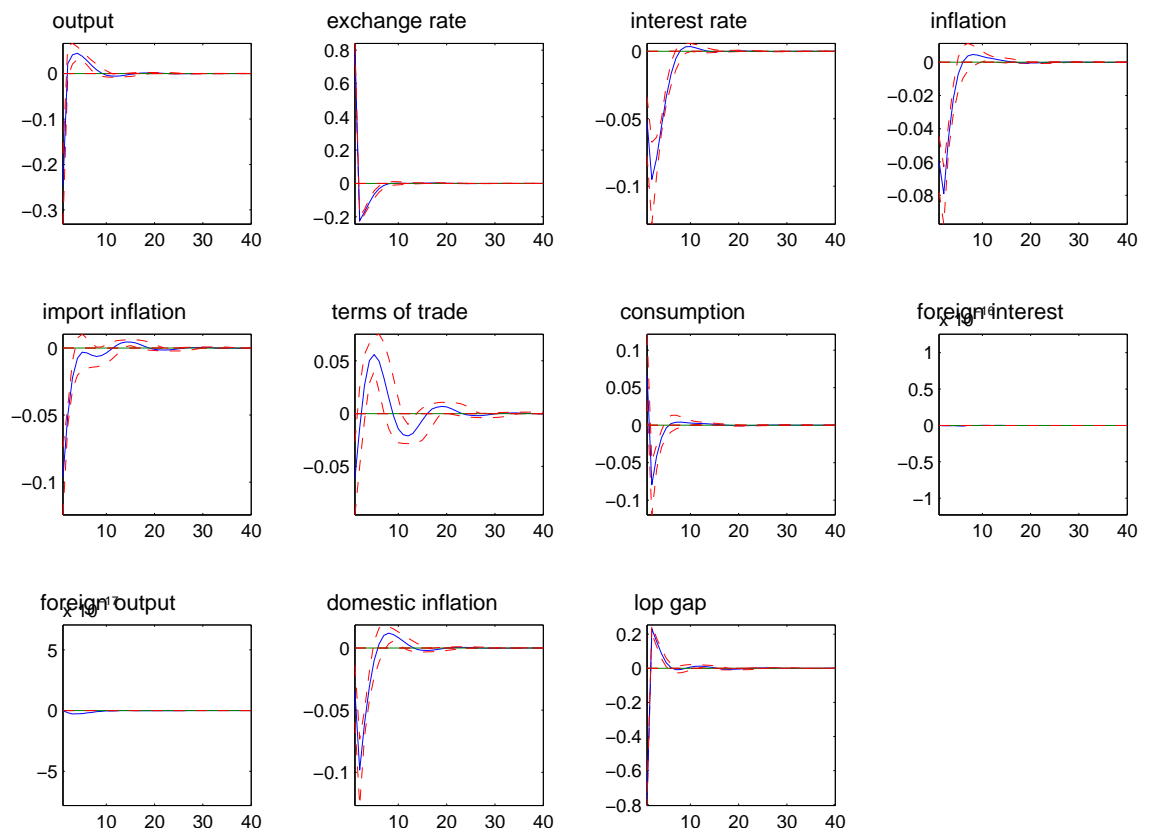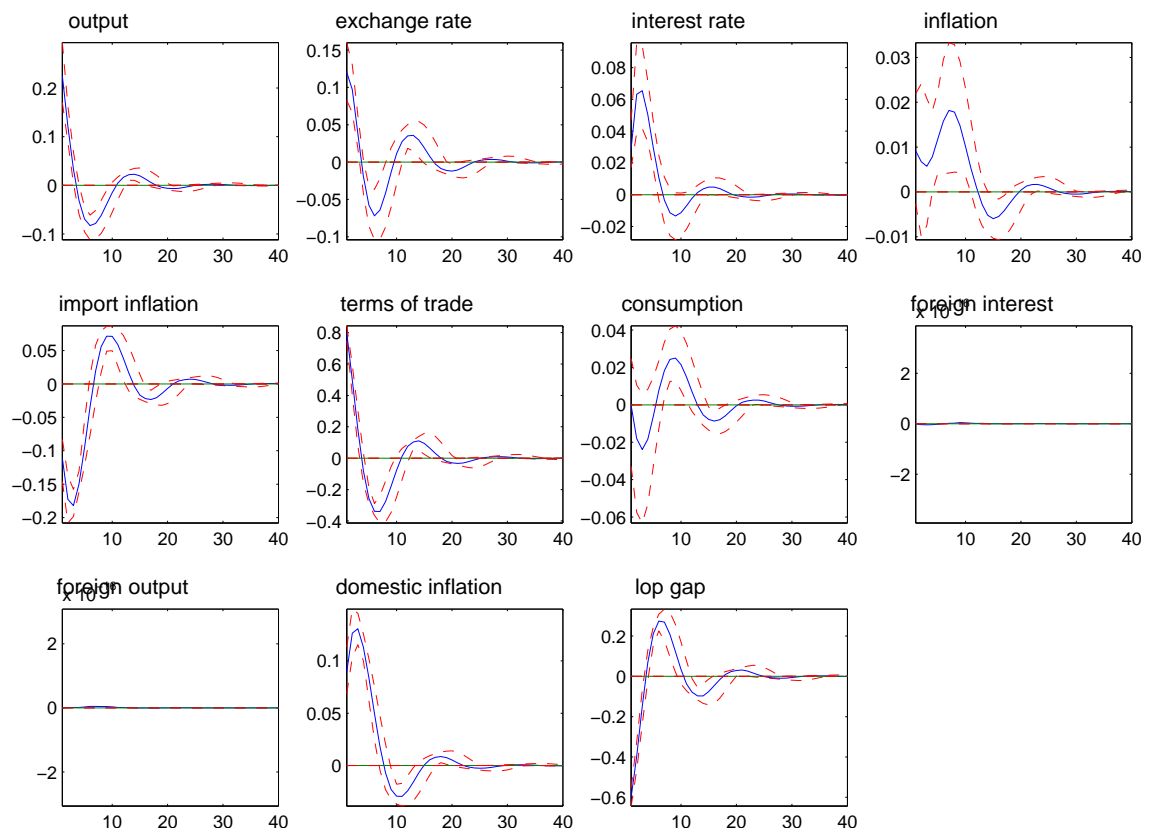**Impulse response functions from one unit of domestic inflation innovation.**

Figure 5

Impulse response functions from one unit of interest rate innovation.

Figure 6
Impulse response functions from one unit of exchange rate innovation.

Figure 7

Impulse response functions from one unit of competitiveness innovation.

Appendix

A The linearized model

1. *Law of one price* gap

$$\psi_t = -[q_t + (1 - \alpha)s_t]$$

2. Terms of trade with measurement error:

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} + \nu_t^s$$

3. Uncovered interest parity condition with a risk premium shock:

$$\Delta E_t q_{t+1} = -\{(r_t - E_t\pi_{t+1}) - (r_t^* - E_t\pi_{t+1}^*)\} + \nu_t^q$$

4. Domestic inflation:

$$\pi_{H,t} = \beta(1 - \theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \lambda_H mc_t + \nu_t^{\pi_H}$$

5. Import inflation:

$$\pi_{F,t} = \beta(1 - \theta_F)E_t\pi_{H,t+1} + \theta_F\pi_{F,t-1} + \lambda_F\psi_t + \nu_t^{\pi_F}$$

6. Overall inflation:

$$\pi_t = (1 - \alpha)\pi_{H,t} + \alpha\pi_{F,t}$$

7. Firm's marginal cost:

$$mc_t = \frac{\sigma}{1 - h}(c_t - hc_{t-1}) + \varphi y_t + \alpha s_t - (1 + \varphi)a_t$$

8. Consumption Euler equation:

$$c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - \frac{1 - h}{\sigma}(r_t - E_t\pi_{t+1})$$

9. International risk sharing condition:

$$c_t - hc_{t-1} = y_t^* - hy_{t-1}^* - \frac{1 - h}{\sigma}q_t$$

10. Goods market clearing condition:

$$y_t = (2 - \alpha)\alpha\eta s_t + (1 - \alpha)c_t + \alpha\eta\psi_t + \alpha y_t^*$$

11. Reaction function:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\phi_1 \pi_t + \phi 2 \Delta y_t) + \nu_t^r$$

12. Exogenous processes:

$$a_t = \rho_a a_{t-1} + \nu_t^a$$
$$y_t^* = \lambda_1 y^* + \nu_t^{y^*}$$
$$(r_t^* - E_t \pi_{t+1}^*) = \rho_{r^*}(r_{t-1}^* - \pi_t^*) + \nu_t^{r^*}$$

B Data description

- Domestic output (y_t) is (linear-) detrended, seasonally adjusted real GDP per capita for New Zealand

- Overall inflation (π_t) is the annual growth rate in consumer price index (CPI) for New Zealand

- Import inflation $(\pi_{F,t})$ is the annual growth rate in the import deflator for New Zealand

- Nominal interest rate (r_t) is the 90-day Bank Bill rate for New Zealand

- Competitive price index (s_t) is calculated by taking the ratio of the (log) foreign (80 percent US and 20 percent Australia) weighted CPI and the (log) domestic GDP excluding import deflator

- Real exchange rate (q_t) is the log of weighted real exchange rate between the US (80 percent) and Australia (20 percent)

- Foreign output (y_t^*) is the weighted average of US (80 percent) and Australia (20 percent) detrended log real GDP per capita.

- Foreign real interest rate $(\bar{r}_t^*)$ is the weighted average of US (80 percent) and Australia (20 percent) short term real interest rates.

Table 1
Prior distributions

Parameter	Domain	Density	Mean	Variance	95% Interval
h	$[0, 1]$	Beta	0.70	0.20	[0.321, 0.965]
σ	$\Re^+$	Normal	1.00	0.25	[0.589, 1.411]
η	$\Re^+$	Gamma	1.00	0.30	[0.563, 1.539]
φ	$\Re^+$	Gamma	1.00	0.30	[0.563, 1.539]
θ_H	$[0, 1]$	Beta	0.50	0.25	[0.097, 0.903]
θ_F	$[0, 1]$	Beta	0.50	0.25	[0.097, 0.903]
δ	$[0, 1]$	Beta	0.50	0.20	[0.178, 0.828]
ϕ_1	$\Re^+$	Gamma	1.50	0.25	[1.113, 1.933]
ϕ_2	$\Re^+$	Gamma	0.25	0.10	[0.111, 0.434]
ρ_a	$[0, 1]$	Beta	0.50	0.20	[0.178, 0.828]
ρ_{r*}	$[0, 1]$	Beta	0.50	0.20	[0.178, 0.828]
λ_1	$[0, 1]$	Beta	0.50	0.20	[0.178, 0.828]
σ_s	$\Re^+$	InvGamma	2	∞	[0, ∞]
σ_q	$\Re^+$	InvGamma	2	∞	[0, ∞]
σ_r	$\Re^+$	InvGamma	2	∞	[0, ∞]
σ_a	$\Re^+$	InvGamma	2	∞	[0, ∞]
σ_{π_H}	$\Re^+$	InvGamma	2	∞	[0, ∞]
σ_{π_F}	$\Re^+$	InvGamma	2	∞	[0, ∞]
σ_{r*}	$\Re^+$	InvGamma	2	∞	[0, ∞]
σ_{y*}	$\Re^+$	InvGamma	2	∞	[0, ∞]

* The parameters α and β were fixed at 0.4 and 0.99 respectively.

Table 2

MCMC diagnostics tests based on two 1.5 million chains

	Post Mean	NSE[1]	P-Value[2]	B-G[3]	Auto(1)[4]	Auto(5)	Auto(10)	Auto(50)
h	0.924	0.001	0.491	1.001	0.946	0.842	0.767	0.458
σ	0.390	0.005	0.676	1.000	0.987	0.939	0.884	0.586
η	0.850	0.010	0.747	1.000	0.997	0.987	0.975	0.892
ϕ	1.828	0.086	0.531	1.009	1.000	0.999	0.998	0.990
θ_H	0.753	0.003	0.483	1.007	0.982	0.930	0.889	0.772
θ_F	0.724	0.001	0.670	1.000	0.963	0.843	0.735	0.429
ϕ_1	1.445	0.005	0.618	1.001	0.995	0.974	0.950	0.791
ϕ_2	0.413	0.004	0.766	1.000	0.994	0.970	0.941	0.758
ρ_r	0.724	0.002	0.775	1.000	0.989	0.948	0.903	0.668
ρ_{rst}	0.832	0.002	0.986	1.000	0.992	0.961	0.924	0.687
ρ_a	0.978	0.000	0.950	1.000	0.923	0.725	0.584	0.224
λ_1	0.785	0.004	0.933	1.000	0.998	0.990	0.980	0.906
σ_a	0.801	0.014	0.407	1.003	0.970	0.887	0.821	0.578
σ_s	9.164	0.042	0.812	1.000	0.998	0.991	0.981	0.912
σ_q	6.068	0.067	0.944	1.000	0.998	0.991	0.982	0.917
σ_{π_H}	1.566	0.003	0.534	1.000	0.955	0.800	0.648	0.171
σ_{π_F}	3.394	0.018	0.896	1.000	0.991	0.958	0.919	0.661
σ_r	0.771	0.002	0.773	1.000	0.991	0.955	0.912	0.645
σ_{y^*}	0.695	0.002	0.649	1.000	0.987	0.939	0.881	0.546
σ_{r^*}	0.538	0.001	0.854	1.000	0.981	0.912	0.833	0.441

1. NSE is the Numeric Standard Error as defined in (Geweke 1999) .

2. The P-Value refers to test of two means generated from two independent chains, the test statistics is with $L = 0.08$, see (Geweke 1999).

3. Univariate "shrink factor" for monitoring the between and within chain variance, see (Brooks and Gelman 1998).

4. Autocorrelation functions at lag 1, 5, 10 and 50 respectively.

Table 3
Posterior estimates using 1.5 million Markov chain draws

Parameter	Prior mean	Post. median	Post. 95% interval
h	0.70	0.92	[0.88, 0.96]
σ	1.00	0.39	[0.19, 0.68]
η	1.00	0.85	[0.58, 1.16]
φ	1.00	1.83	[0.83, 3.14]
θ_H	0.50	0.75	[0.71, 0.80]
θ_F	0.50	0.72	[0.69, 0.76]
ϕ_1	1.50	1.44	[1.27, 1.67]
ϕ_2	0.25	0.41	[0.28, 0.58]
δ	0.50	0.72	[0.65, 0.79]
ρ_{r^*}	0.50	0.83	[0.76, 0.89]
ρ_a	0.50	0.98	[0.95, 0.99]
λ_1	0.50	0.78	[0.67, 0.87]
σ_a	$[0,\infty]$	0.80	[0.49, 1.27]
σ_s	$[0,\infty]$	9.16	[7.63, 11.08]
σ_q	$[0,\infty]$	6.07	[4.68 , 8.10]
σ_{π_H}	$[0,\infty]$	1.57	[1.27, 1.94]
σ_{π_F}	$[0,\infty]$	3.39	[2.70, 4.30]
σ_r	$[0,\infty]$	0.77	[0.64, 0.94]
σ_{y^*}	$[0,\infty]$	0.69	[0.58, 0.84]
σ_{r^*}	$[0,\infty]$	0.54	[0.44, 0.66]

* The parameters α and β were fixed at 0.4 and 0.99 respectively.