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Search-and-matching frictions and labor market dynamics in Latvia*

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Abstract

This paper examines, in an estimated, full-fledged New Keynesian DSGE model with Nash wage bargaining, sticky wage and high value of leisure akin to Christiano, Trabandt and Walentin (2011), whether search-and-matching frictions in labor market can explain aggregate labor market dynamics in Latvia. If vacancies are not observed, the model can, to a reasonable degree, generate realistic variance and dynamics of unemployment, and the correlation between unemployment and (latent) vacancies, but at the expense of too volatile vacancies. As a by-product, one-quarter ahead forecasts of hours worked and GDP exhibit less excess volatility and thus are more precise, compared to a model without search-and-matching frictions. However, if both unemployment and vacancies are observed and a shock to the matching efficiency is allowed for, then the cyclical behavior of forecasted vacancies - and the correlation between unemployment and vacancies - tends to counter the data (to the benefit of better fit of vacancies' volatility), and the smoothed matching efficiency is counter-intuitively counter-cyclical. Hence the model cannot fit the three statistics - variance of unemployment and vacancies, and the correlation between the two - simultaneously.

Keywords: DSGE model, unemployment, small open economy, Bayesian estimation, currency union, forecasting
JEL code: E0, E3, F0, F4

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1 Introduction

The standard business cycle approach of modeling labor markets without explicit unemployment (as in Erceg, Henderson and Levin, 2000, henceforth EHL) has its drawbacks. Its main drawback is that it has no implications for unemployment (so-called 'extensive margin' of labor supply) and thus the variation of total hours worked is attributed solely to the variation in hours per employee ('intensive margin'). Second, it also tends to induce too little persistence in hours worked (see e.g. Buss, 2014).

In reality, much of the variation in total hours worked is generated by the extensive margin of labor supply. To quantify that, this paper applies a simple data variance decomposition to Latvian data for the period of 2002Q1 - 2012Q4, as detailed in the next section. Though the data are noisy and thus the decomposition is rough, according to it, more than a half of the variation in total hours worked is explained by the variation in the number of employees. The two employment margins have different economic policy implications, thus it is useful to distinguish between the two in economic analysis.

Search-and-matching theory has become the most widespread economic theory of labor market since Merz (1995) and Andolfatto (1996) integrated the original Diamond-Mortensen-Pissarides framework into a standard general equilibrium model. The merit of search-and-matching models is due to the fact that market-clearing real business cycle models were unable to explain unemployment and the co-existence of unfilled vacancies and unemployed workers.

Nevertheless, the work of Shimer (2005) started a yet lively discussion of whether this theory can fit the data. Shimer (2005) concludes that the model in its basic form cannot fit the second moments of unemployment and vacancies. Many types of corrections to the model have been proposed, such as sticky wage (Hall, 2005a), on-the-job search (Mortensen and Nagypal, 2007), high value of leisure (Hagedorn and Manovskii, 2008), and alternating-offer wage bargaining (Hall and Milgrom, 2008). Many of the proposals are united by the mechanism they affect the 'fundamental surplus' - the fraction of firm profits allocated to create vacancies - which is the source of amplification and persistence of unemployment in these models (Ljungqvist and Sargent, 2015).

Much of the lengthy literature is devoted to either calibrated models or to study the US data. Rarely the models are estimated using non-US data, and even more so, using a full-fledged model. This paper adds to the literature by studying the ability of a richly specified New Keynesian model with search-and-matching frictions to fit the key moments of unemployment and vacancies, particularly for a non-US country. The model in this paper is closest to Christiano, Trabandt and Walentin (2011, CTW). The innovation of this paper compared to CTW is that it i) adjusts the model to a member country of a currency union, ii) estimates the model to Latvia, and most importantly iii) studies the model's ability to fit both unemployment and vacancies simultaneously, not in isolation. This is done by using two specifications of the model: in one specification the vacancies data are unobserved and the matching function is calibrated, resembling the CTW specification, but with implications to (latent) vacancies; in another specification, vacancies data are observed, the matching function is estimated, including the shock to the matching efficiency.

This paper confirms CTW's findings that the model can fit unemployment well. But the paper goes a step further and finds that CTW's favored specification can fit also the correlation between unemployment and vacancies rather well; however, the decent fit of the above two statistics comes at a high cost of having vacancies' standard deviation multiple (specifically, 2.9) times higher than in the data.

However, if both unemployment and vacancies are observed and a shock to the matching efficiency is allowed for, then the cyclical behavior of forecasted vacancies - and the correlation between unemployment and vacancies - tends to counter the data (to the benefit of better fit of vacancies' volatility), and the smoothed matching efficiency counter-intuitively is counter-cyclical. Hence the model cannot fit the three statistics - variance of unemployment and vacancies, and the correlation between the two - simultaneously.

As by-product of adding the search-and-matching frictions to a model, one-quarter ahead forecasts of hours worked and GDP exhibit less excess volatility and thus are more precise, compared to a model without search-and-matching frictions.

There are few studies, that use estimated, full-fledged DSGE models with search-and-matching frictions, and study the fit of the data moments. Christiano, Eichenbaum and Trabandt (2013, CET) employ alternating-offer wage bargaining (AOB) mechanism within an estimated New Keynesian model for the US data and find that the model fits the key data moments well. However, they find that the same model but with Nash wage bargaining, though inferior, yields a close fit of the key data moments compared to the AOB specification. That result differs from ours for Latvia. Moreover, CET do not estimate the shock to matching efficiency. The differing results between the US and Latvia call for more studies across economies. Yet it is instructive to test the AOB model to Latvian data.

The paper is structured as follows. Section 2 overviews the model. Section 3 describes the estimation procedure and the results. Section 4 concludes. Appendices contain more information about the model, its estimation and the results.

2 The model in brief

This paper adds the labor market frictions block of Christiano, Trabandt and Walentin (2011, henceforth CTW) to the model with financial frictions block of Buss (2014), which serves as a benchmark.

Since the model is almost a replica of CTW, this section is a brief introduction to the model, whereas its formal description is relegated to Appendix C. The only noticeable difference between the CTW model and this one is in the behavior of monetary authority which is modeled as a currency union in this paper.

2.1 Benchmark financial frictions model

The financial frictions model consists of the core block and the financial frictions add-in.

The **core block** builds on Christiano, Eichenbaum and Evans (2005) and Adolfson, Laseen, Linde and Villani (2008). The three final goods: consumption, investment and exports, are produced by combining the domestic homogeneous good with specific imported inputs for each type of final good. Specialized domestic importers purchase a homogeneous foreign good, which they turn into a specialized input and sell to domestic import retailers. There are three types of import retailers. One uses the specialized import goods to create a homogeneous good used as an input into the production of specialized exports. Another uses the specialized import goods to create an input used in the production of investment goods. The third type uses specialized imports to produce a homogeneous input used in the production of consumption goods. Exports involve a Dixit-Stiglitz (Dixit and Stiglitz, 1977) continuum of exporters, each of which is a monopolist that produces a specialized export good. Each monopolist produces its export good using a homogeneous domestically produced good and a homogeneous good derived from imports. The homogeneous domestic good is produced by a competitive, representative firm. The domestic good is allocated among the i) government consumption (which consists entirely of the domestic good) and the production of ii) consumption, iii) investment, and iv) export goods. A part of the domestic good is lost due to the real friction in the model economy due to investment adjustment and capital utilization costs.

Households maximize expected utility from a discounted stream of consumption (subject to habit) and leisure. In the core block, the households own the economy's stock of physical capital. They determine the rate at which the capital stock is accumulated and the rate at which it is utilized. The households also own the stock of net foreign assets and determine its rate of accumulation.

The monetary policy is conducted as a hard peg of the domestic nominal interest rate to the foreign nominal interest rate.

The government spending grows exogenously. The taxes in the model economy are: capital tax, payroll tax, consumption tax, labor income tax, and a bond tax. Any difference between government spending and tax revenue is offset by lump-sum transfers.

The foreign economy is modeled as a structural vector autoregression (henceforth, SVAR) in foreign output, inflation, nominal interest rate and technology growth. The model economy has two sources of exogenous growth: neutral technology growth and investment-specific technology growth.

The **financial frictions add-in** attaches the Bernanke, Gertler and Gilchrist (1999, henceforth BGG) financial frictions to the above core block. Financial frictions reflect that borrowers and lenders are different people, and that they have different information. Thus the model introduces 'entrepreneurs' - agents who have a special skill in the operation and management of capital. Their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support, by borrowing additional funds. There is financial friction because the management of capital is risky, i.e. entrepreneurs can go bankrupt, and only the entrepreneurs costlessly observe their own idiosyncratic productivity.

In this block, the households deposit money in banks. The interest rate that households receive is nominally non state-contingent.[1] The banks then lend funds to entrepreneurs using a standard nominal debt contract, which is optimal given the asymmetric information.[2] The amount that banks are willing to lend to an entrepreneur under the debt contract is a function of the entrepreneurial net worth. This is how balance sheet

[1]These nominal contracts give rise to wealth effects of unexpected changes in the price level, as emphasized by Fisher (1933). E.g., when a shock occurs which drives the price level down, households receive a wealth transfer. This transfer is taken from entrepreneurs whose net worth is thereby reduced. With tightening of their balance sheets, the ability of entrepreneurs to invest is reduced, and this generates an economic slowdown.

[2]Namely, the equilibrium debt contract maximizes the expected entrepreneurial welfare, subject to the zero profit condition on banks and the specified return on household bank liabilities.

constraints enter the model. When a shock occurs that reduces the value of entrepreneurs' assets, this cuts into their ability to borrow. As a result, entrepreneurs acquire less capital and this translates into a reduction in investment and leads to a slowdown in the economy. Although individual entrepreneurs are risky, banks are not.

The financial frictions block brings two new endogenous variables, one related to the interest rate paid by entrepreneurs and the other - to their net worth. There are also two new shocks, one to idiosyncratic uncertainty and the other - to entrepreneurial wealth.

2.2 Full model with labor market frictions block

I apply a simple data variance decomposition to Latvian data for the period of 2002Q1 - 2012Q4:[3]

$$Var(H_t) = Var(\varsigma_t) + Var(L_t) + 2Covar(\varsigma_t, L_t)$$

where H_t denotes total hours worked, ς_t - hours per employee, and L_t - the number of people employed, Var - variance, $Covar$ - covariance. H_t and L_t are in per capita terms, H_t and ϱ_t are normalized by the average hours worked, and all series are logged. Though the data are noisy and thus the decomposition is rough, according to it, about 58% of the variation in total hours is explained by the variation in employment, 28% is attributed to the variation in hours per employee, and about 14% - to the covariance term.

Therefore, this paper adds the labor market search and matching framework of Mortensen and Pissarides (1994), Hall (2005a,b) and Shimer (2005, 2012), with Taylor-type nominal wage rigidity as modeled in CTW, to the benchmark financial frictions model of Buss (2014). A key feature of this model is that there are wage-setting frictions but they do not have a direct impact on on-going worker-employer relations as long as these are mutually beneficial[4]. However, wage-setting frictions have an impact on the effort of an employer in recruiting new employees[5]. Accordingly, the setup is not vulnerable to the Barro (1977) critique that wages cannot be allocational in on-going employer-employee relationships. Also, the intensive margin of labor supply is allowed, as well as the endogenous separation of employees from their jobs.

As in the benchmark financial frictions model, there is the Dixit-Stiglitz specification of homogeneous goods production. A representative, competitive retail firm aggregates differentiated intermediate goods into a homogeneous good. Intermediate goods are supplied by monopolists who hire labor and capital services in competitive factor markets. The intermediate good firms are assumed to be subject to the same Calvo price setting frictions as in the benchmark model.

The search and matching framework dispenses with the specialized labor services abstraction and the accompanying monopoly power in the benchmark model. Labor services are instead supplied by 'employment agencies' - a modeling construct best viewed

[3]The sample period is constrained by the data availability.

[4]That is, the existence of the nominal wage frictions do not imply that the employer-employee relations are enforced upon them, since they can separate if their relationship is not beneficial.

[5]The Nash wage depends on the relative bargaining power between the employer and the employee. The smaller is the relative bargaining power of the employee, the smaller is the Nash wage and thus the greater incentive to recruit new employees.

as a goods producing firm's human resource division - to the homogeneous labor market where they are bought by the intermediate goods producers.[6]

Each employment agency retains a large number of workers. Each employment agency is permanently allocated to one of $N = 4$ different equal-sized cohorts. Cohorts are differentiated according to the period (quarter) in which they renegotiate their wage. The nominal wage paid to an individual worker is determined by Nash bargaining, which occurs once every N periods.[7] Since there is an equal number of agencies in each cohort, $1/N$ of the agencies bargain in each period. The intensity of labor effort is determined efficiently by equating the worker's marginal cost to the agency's marginal benefit. The assumption of efficient provision of labor on the intensive margin without any direct link to the sticky wage allows for a high frequency disconnect between wages and hours worked. Fundamentally, this model reflects that labor is not supplied on a spot market but within long-term relationships.

The events during the period in an employment agency take place in the following order. At the beginning of the period an exogenously determined fraction of workers is randomly selected to separate from the agency and go into unemployment. Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the agency in the previous period. Then, the economy's aggregate shocks are realized. After that, each agency's nominal wage rate is set. When a new wage is set, it evolves over the subsequent $N-1$ periods. The wage negotiated in a given period covers all workers employed at an agency for each of the subsequent $N-1$ periods, even those that will arrive later. Next, each worker draws an idiosyncratic productivity shock. A cutoff level of productivity is determined, and workers with lower productivity are laid off.[8] After the endogenous layoff decision, the employment agency posts vacancies and the intensive margin of labor supply is chosen efficiently by equating the marginal value of labor services to the employment agency with the marginal cost of providing it by the household. At this point the employment agency supplies labor to the labor market.

The explicit description of the model is relegated to Appendix C.

[6]The change leaves the equilibrium conditions associated with the production of the homogeneous good unaffected. Key labor market activities - vacancy postings, layoffs, labor bargaining, setting the intensity of labor effort - are all carried out inside the employment agencies. Each household is composed of many workers, each of which is in the labor force. A worker begins the period either unemployed or employed with a particular agency with a probability that is proportional to the efforts made by the agency to attract workers. Workers are separated from employment agencies either exogenously, or because they are actively cut. Workers pass back and forth between unemployment and employment - there are no agency to agency transitions.

[7]The bargaining arrangement is atomistic, so that each worker bargains separately with a representative of the employment agency.

[8]This is the endogenous part of the separation, as opposed to the exogenous separation mentioned at the beginning of the paragraph. From a technical point of view this modeling is symmetric to the modeling of entrepreneurial idiosyncratic risk and bankruptcy. Two mechanisms are considered by which the cutoff is determined. One is based on the total surplus of a given worker, and the other is based purely on the employment agency's interest.

3 Estimation and results

The time unit is a quarter. A subset of model's parameters is calibrated and the rest are estimated using the data for Latvia (domestic part) and the euro area (foreign part). To save space, the calibration details are relegated to Appendix A.

The model is estimated with Bayesian techniques. Two versions of the model will be discussed. In one version, the model is fed with 19 observables including the (quarterly growth rates of) unemployment rate, but vacancies data are not observed, and the parameters in the matching function are calibrated. In another version, the model is fed with 20 observables, including the data on both unemployment and vacancies. In the latter version, the parameters in matching function are estimated, including the shock to the matching efficiency. Prior-posterior information is relegated to Appendix A.

3.1 Vacancies unobserved

If vacancies are not observed, then the parameters of matching function are calibrated. The Cobb-Douglas matching function is of the form

$$m_t = \sigma_m (1 - L_t)^{\sigma} v_t^{1-\sigma} \tag{3.1}$$

where m_t denotes the total matches, L_t - the fraction of employed, v_t - total vacancies, σ_m - level parameter, σ - unemployment share. The particular calibration is $\sigma_m = 0.4$ and $\sigma = 0.5$ (Table 1). This calibration together with the rest of model parameter values reported in Appendix A, yields the following results.

Description	Vacancies unobservable	Vacancies observable						
	Calibrated	Prior			Posterior		HPD int.	
		Distr.	Mean	st.d.	Mean	st.d.	10%	90%
Unemployment share σ	0.500	β	0.5	0.05	0.373	0.017	0.326	0.417
Level parameter σ_m	0.400	β	0.4	0.05	0.394	0.024	0.332	0.442
Shock standard deviations								
Matching efficiency	0	Inv-Γ	0.1	inf	12.810	1.429	10.925	14.624

Table 1: Matching function parameters.

Model and data moments The model-implied standard deviation of first differenced unemployment rate is 10.35 versus 9.75 in the sample data (Table 2). The second-moment fit is closer than that for the U.S. reported by Shimer (2005). This is due to at least two sources: i) the assumed wage stickiness (as emphasized by Hall, 2005a) a la Taylor, and ii) the high estimated replacement ratio (0.80 at the posterior mean), as emphasized by Hagedorn and Manovskii (2008).

	corr (Δu,Δv)	st.d. Δu	st.d. Δv
data	**-0.54**,[-0.68 -0.35]	**9.75**,[8.37 11.68]	**16.04**,[13.76 19.21]
model: no vacancies	**-0.40**,[-0.42 -0.37]	**10.35**,[10.15 10.55]	**46.62**,[45.72 47.55]
model: with vacancies	**-0.30**,[-0.32 -0.27]	**9.48**,[9.29 9.67]	**36.19**,[35.50 36.91]

Table 2: Data and model moments.

Note: 1) The statistics for data are calculated using 71 obs. long samples, and those for the two models are calculated using 5000 obs. long simulated data at the posterior mean. 2) 95% confidence interval in brackets.

The model-implied correlation between first differences of unemployment and vacancies is also decent (-0.40) though lower than in the sample data (-0.54).

However, the above two moments are fitted at the cost of too volatile vacancies: model-implied standard deviation of first differenced vacancies is 2.9 times that in the sample data (46.6 vs 16.0).[9]

Conditional variance decomposition The conditional variance decomposition indicates that 3/4 of the variance of first differenced unemployment rate at 8 quarters forecast horizon are explained by the markup shock to imports for exports (35.9%), the markup shock to imports for investment (18.1%), the labor preference shock (13.3%), and the stationary technology shock (5.4%). While 4/5 of hours per employee are explained by the labor preference shock alone (Table 3, last two columns).

[9] The volatility of vacancies is not reduced substantially when the share of cost of vacancy creation in the total cost of meeting a worker is raised from zero to 20%.

	Description	model	R	π^c	GDP	C	I	$\frac{NX}{GDP}$	H	w	q	N	Spread	$\frac{H}{L}$	U
ϵ_t	Stationary technology	finfric	0.0	0.6	0.7	0.1	0.0	0.5	8.6	0.3	0.6	0.1	0.1		
		full	0.1	4.2	11.2	1.0	0.3	3.4	8.1	3.3	3.7	0.8	0.5	1.2	8.0
Υ_t	MEI	finfric	0.1	0.0	1.8	0.1	19.2	3.4	3.2	0.2	0.0	12.7	12.2		
		full	0.1	0.3	2.2	0.1	33.2	7.1	2.6	1.8	0.3	10.4	11.6	0.2	3.4
ζ_t^c	Consumption prefs	finfric	0.2	0.0	7.1	78.7	0.1	14.8	5.6	0.0	0.0	0.1	0.1		
		full	0.9	1.3	6.5	85.6	0.3	29.1	7.0	4.9	1.2	0.5	0.4	4.3	7.3
ζ_t^h	Labor prefs	finfric	0.1	10.1	5.8	4.0	1.0	6.6	8.0	51.7	9.3	2.0	0.6		
		full	0.1	3.3	9.7	1.3	0.2	2.5	26.1	16.0	3.0	0.6	0.3	89.7	19.0
g_t	Govt spending	finfric	0.0	0.0	8.2	0.0	0.0	0.1	6.6	0.0	0.0	0.0	0.0		
		full	0.0	0.2	2.7	0.0	0.0	0.5	2.0	1.1	0.2	0.1	0.1	0.2	2.0
τ_t^d	Markup, domestic	finfric	0.0	22.7	2.2	0.1	0.1	0.3	1.8	33.0	20.8	0.5	0.1		
		full	0.0	20.6	1.4	0.1	0.1	0.1	1.4	44.0	18.3	0.7	0.0	0.1	2.8
τ_t^x	Markup, exports	finfric	0.0	0.0	2.3	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.0		
		full	0.0	0.5	3.0	0.0	0.1	0.1	2.7	2.5	0.5	0.2	0.1	0.3	4.3
τ_t^{mc}	Markup, imp. for cons.	finfric	0.0	59.1	4.1	0.1	0.0	1.1	3.2	3.9	54.1	0.1	0.0		
		full	0.0	56.7	1.9	0.0	0.1	0.5	1.5	2.5	50.5	0.2	0.1	0.1	2.5
τ_t^{mi}	Markup, imp. for inv.	finfric	0.1	0.3	23.3	0.0	5.4	6.0	34.4	0.1	0.3	7.9	6.1		
		full	0.1	2.5	9.0	0.0	12.6	7.5	16.2	2.3	2.2	19.4	17.3	1.0	12.0
τ_t^{mx}	Markup, imp. for exp.	finfric	0.1	0.0	28.4	0.1	0.1	6.2	22.8	0.1	0.0	0.2	0.1		
		full	0.1	1.4	35.6	0.0	0.2	4.6	24.9	7.1	1.3	0.4	0.5	1.5	23.3
γ_t	Entrepreneurial wealth	finfric	0.7	0.6	10.1	0.2	58.1	38.9	1.1	0.6	0.5	62.4	77.3		
		full	0.8	0.5	8.7	0.1	37.5	26.5	0.6	1.2	0.4	52.5	65.1	0.1	0.9
$\tilde{\phi}_t$	Country risk premium	finfric	91.8	0.4	2.2	5.5	8.7	17.8	0.8	3.1	0.4	9.4	1.7		
		full	90.1	0.4	1.6	1.7	8.3	13.7	1.0	3.6	0.3	8.6	1.6	1.0	4.3
$\mu_{z,t}$	Unit-root technology	finfric	1.7	0.0	0.2	0.0	0.1	1.4	0.0	0.2	0.2	0.1	0.0		
		full	1.8	0.0	0.4	0.0	0.1	1.5	0.0	0.6	0.2	0.0	0.0	0.1	0.0
$\epsilon_{R^*,t}$	Foreign interest rate	finfric	1.6	0.1	0.1	0.3	0.3	0.7	0.0	0.1	0.0	0.2	0.0		
		full	1.8	0.1	0.1	0.2	0.3	0.9	0.0	0.2	0.0	0.3	0.1	0.1	0.2
$\epsilon_{y^*,t}$	Foreign output	finfric	3.6	0.1	0.0	0.8	0.2	2.1	0.0	0.3	0.2	0.0	0.0		
		full	4.0	0.2	0.1	0.5	0.2	2.2	0.1	0.2	0.3	0.0	0.1	0.2	0.1
$\epsilon_{\pi^*,t}$	Foreign inflation	finfric	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0		
		full	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0
	5 foreign*	finfric	98.7	0.6	2.6	6.6	9.3	22.0	0.8	3.6	0.9	9.7	1.8		
		full	97.7	0.7	2.2	2.5	8.9	18.3	1.1	4.5	0.9	9.0	1.7	1.3	4.6
	All foreign**	finfric	98.9	60.1	60.7	6.8	14.9	35.3	63.1	7.8	55.3	17.9	8.0		
		full	98.0	61.9	51.7	2.6	21.8	31.0	46.3	18.9	55.4	29.2	19.7	4.2	46.7
	Measurement error	finfric	0.0	5.8	3.4	10.0	6.6	0.0	1.9	6.4	13.5	4.2	1.7		
		full	0.0	7.7	5.9	9.2	6.5	0.0	5.8	8.9	17.6	5.3	2.3	0.0	9.9

Table 3: Conditional variance decomposition (percent) given model parameter uncertainty at 8 quarters forecast horizon; posterior mean.

Note: R - nominal interest rate, π^c - CPI, C - real private consumption, I - real investment, $\frac{NX}{GDP}$ - net exports to GDP ratio, H - total hours worked, w - real wage, q - real exchange rate, N - net worth, Spread - interest rate spread, $\frac{H}{L}$ - hours per employee, U - (first differenced) unemployment rate;
$*$ '5 foreign' is the sum of the foreign stationary shocks, R_t^*, π_t^*, Y_t^*, the country risk premium shock, $\tilde{\phi}_t$, and the world-wide unit root neutral technology shock, $\mu_{z,t}$.
$**$ 'All foreign' includes the above five shocks as well as the markup shocks to imports and exports, i.e. τ_t^{mc}, τ_t^{mi}, τ_t^{mx} and τ_t^x. 'finfric' - benchmark financial frictions model, 'full' - full model with unemployment.

Unemployment in impulse response analysis One of the main benefits of having unemployment in a general equilibrium model is to be able to study the effects of various shock scenarios on unemployment. The below analysis serves as an illustrative example of such analysis.

Since Table 3 shows that the entrepreneurial wealth shock[10] is one of the key drivers of the variance of investment, it is instructive to discuss the impulse response functions (IRF) of this shock. The IRF to the entrepreneurial wealth shock are plotted in Figure 1, which shows that a positive temporary entrepreneurial wealth shock drives up net worth, reduces interest rate spread, and thus increases investment (by about the same percentage change as in net worth); GDP goes up accordingly, and so the real wage and total hours worked. Both exports and imports increase but the latter increases more due to the demand for investment goods, thus net exports to GDP ratio decreases slightly. As a consequence, the net foreign assets to GDP ratio worsens, driving up slight risk premium on the domestic nominal interest rate. Inflation goes down and the real exchange rate depreciates.

The above results are broadly similar across the two models, but the addition of the labor block allows us to study the effects on the labor market: unemployment rate drops, and hours per employee increase.[11]

[10] For example, a shock to the entrepreneur's asset price.

[11] Here and in other IRFs of the full model, the real wage rate jumps after around 4 quarters, and this is the artifact of the Taylor-type modeling of nominal wage rigidity. In particular, wages are renegotiated every 4 quarters, in a staggered way. Therefore, after a shock has occurred, some of the employment agencies are stuck with wages that they set before the shock hits. Depending on how much wage adjustment is needed, the adjustment can be quite vigorous when the "second to last" or "last" employment agency have their turns to set their wages optimally. Such dynamics of the modeled real wage can be considered as implausible and suggest that the Taylor-type frictions may be too strict for the particular sample of Latvian data. Whereas the Taylor-type frictions might be a reasonable approximation of reality in normal times, it appears to fail during the great recession episode when the real (and nominal) wage was rather flexible in Latvia. This evidence calls for revision of the way wage rigidity is modeled.

Meanwhile, the IRF figure contains also the shadow wage, or the marginal product of labor (MPL), i.e. the real wage the entrepreneurs would be willing to pay their workers absent the wage rigidity. in this and other IRFs, the shadow wage reacts more sharply than wages in both the benchmark and the full models. Also, the shadow wage adjusts more rapidly, i.e. its dynamics often dies out within a year, while the rigid wages continue to adjust.

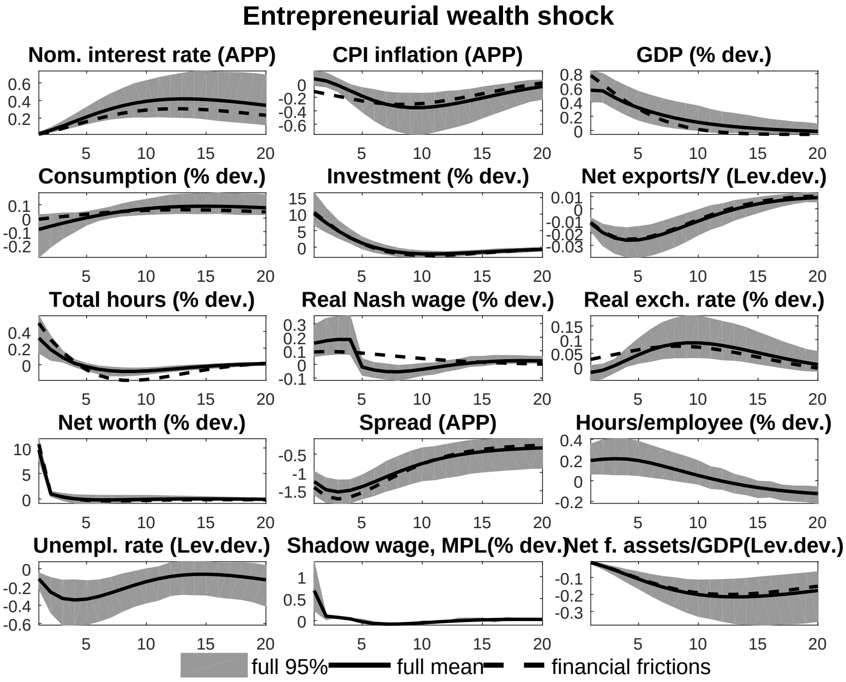

Figure 1: Impulse responses to the entrepreneurial wealth shock.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

Historical shock decomposition Figure 2 shows the decomposition of unemployment rate. The model predicts that the main driving forces of the unemployment during the 2005-boom were the labor and the consumption preference shocks, while during the 2008-recession these same shocks together with the country risk premium and the markup shocks to imports for exports drove unemployment upwards.

The role of the markup shock to imports for exports might require explanation. During 2006-2008 this shock was persistently positive, raising pressure for the substitution of the imported inputs for the domestic inputs in the production of exports, and thus lowering unemployment. However, during the period of 2009-2012, this shock is estimated to be persistently negative. Such a development in relative prices of inputs boosted imports for the production of exports, thus the foreign trade grew substantially, but partly at the expense of a lower growth of the domestic production, thus contributing to a higher unemployment.[12]

Comparing to the results of CTW for Seweden, there are differences in the driving forces of unemployment between the two countries. For Sweden, the entrepreneurial wealth, exports markup and consumption preference shocks drew the unemployment down during the pre-recession period of 2007-2008, while these same shocks contributed much for the reverse process during the great recession.

[12]Having said that, the contribution of the markup shock to imports for exports diminishes if variances of data measurement errors are estimated rather than calibrated to explain 10% of data variances (the results are not reported due to space constraints), hence it is not clear how much of this shock represents a structural shock, and how much - a model misspecification or a data measurement error.

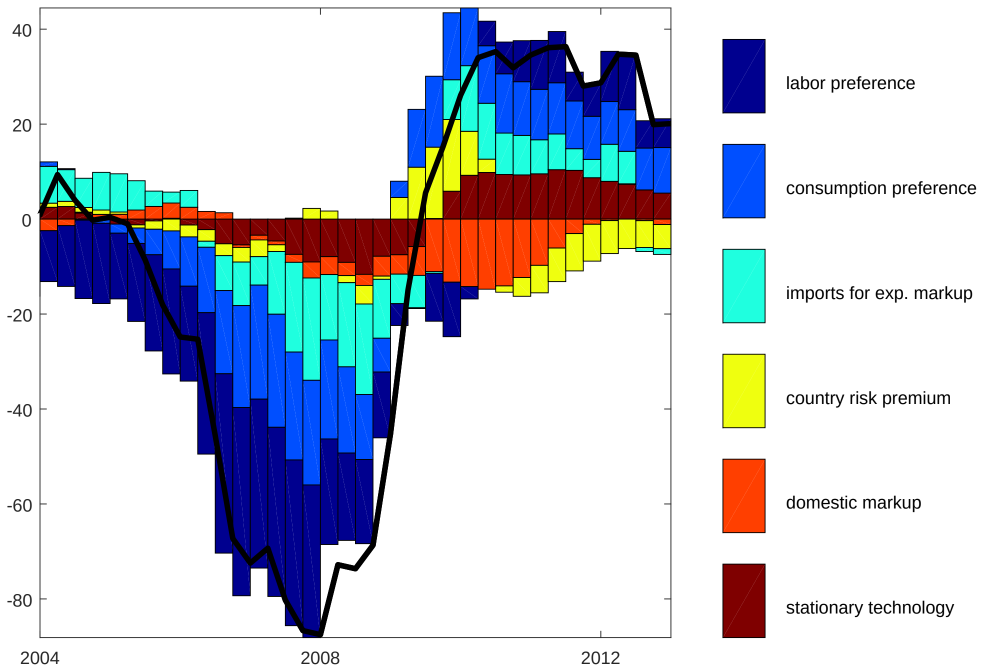

Figure 2: Decomposition of unemployment rate, $1 - L_t$, 2004Q1-2012Q4.

Note: Only the six shocks with the greatest influence shown.

Forecasting performance Figure 3 shows one-quarter ahead forecasts of the full and the benchmark financial frictions models for selected observables.[13]

The introduction of labor block apparently improves the one-step ahead forecasts for total hours worked, and thus also for GDP; the volatility of both of these variables have been reduced. The forecasts of quarterly growth rate of unemployment rate are decent, and those of (latent) quarterly growth rate of vacancies, though highly volatile, have reasonable business cycle dynamics.

[13]These are not true out-of-sample forecasts because the models are calibrated/estimated on the whole sample period 1995Q1-2012Q4. Nevertheless, these figures indicate approximate forecasting performance of the models. Particularly, it is informative to see whether the models tend to yield unbiased forecasts and how the addition of labor block affects forecasts.

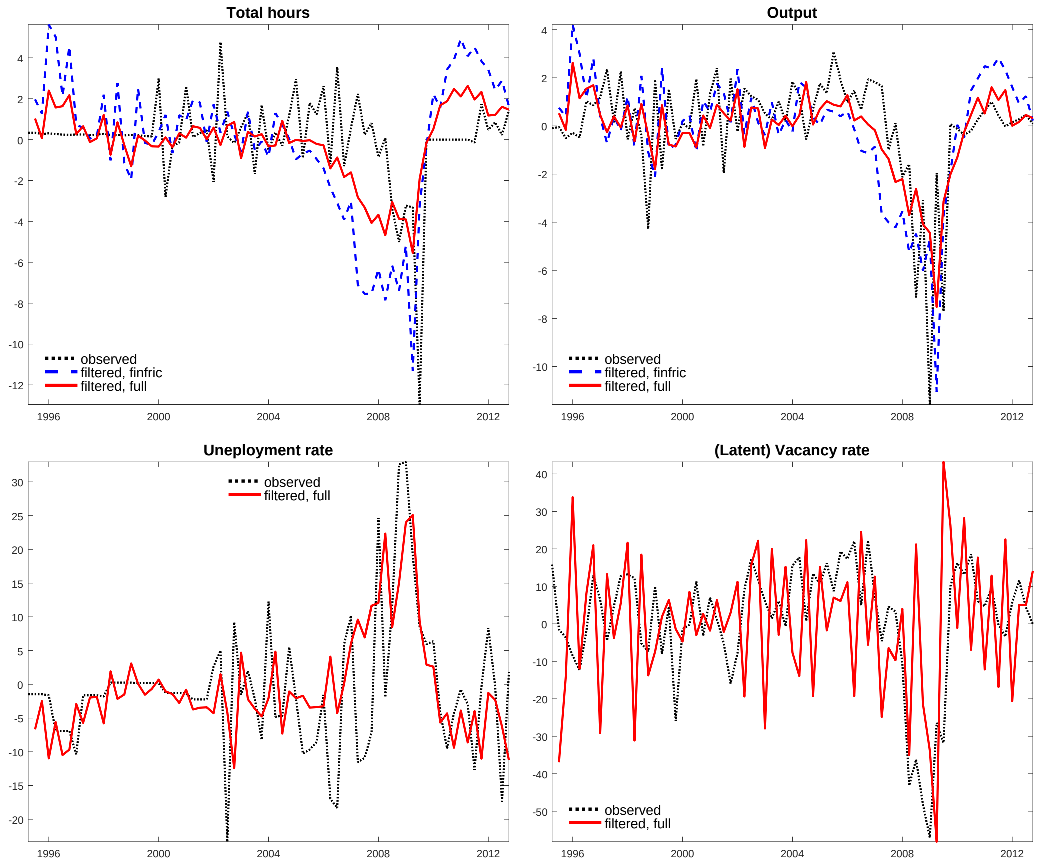

Figure 3: One-step ahead forecasts (selected)

Table 4 reports the forecasting performance of the full and the benchmark models relative to a random walk model (in terms of quarterly growth rates) with respect to predicting CPI inflation and GDP for horizons: one, four, eight and 12 quarters. The table also reports the forecasting performance of a Bayesian SVAR (with the same structure as the foreign SVAR, and with similar priors), since it is often taken as a benchmark in the literature[14].

Table 4 shows that the model forecasts both variables at least as precisely as the random walk model at all horizons considered, and its relative performance improves with higher horizon. Moreover, the full model tends to outperform the benchmark financial frictions model at a one-quarter horizon in GDP forecasting, likely due to the more persistent modeled total hours worked. The performance of the full model is roughly comparable to that of the Bayesian SVAR.

[14] The particular SVAR has some economically implausible estimated parameters, since Latvian GDP, CPI inflation and nominal interest rate data do not possess a stable and economically plausible relationship over the particular sample span.

Model	Distance measure	1Q π^c	1Q Δy	4Q π^c	4Q Δy	8Q π^c	8Q Δy	12Q π^c	12Q Δy
Finfric	RMSE	1.00	0.96	0.78	0.70	0.65	0.64	0.68	0.64
	MAE	0.93	1.15	0.80	0.69	0.70	0.57	0.66	0.60
Full	RMSE	0.93	0.75	0.84	0.72	0.66	0.64	0.68	0.64
	DM p-val	0.078	**0.003**	0.863	0.731	0.617	0.677	0.539	0.657
	MAE	0.88	0.85	0.88	0.67	0.70	0.57	0.66	0.59
	DM p-val	0.135	**0.000**	0.920	0.234	0.464	0.393	0.508	0.198
SVAR	RMSE	0.86	0.72	0.71	0.81	0.62	0.68	0.63	0.66
	MAE	0.89	0.71	0.70	0.77	0.62	0.62	0.58	0.61

Table 4: Forecasting performance.

Note: 1) For RMSE (root mean squared error) and MAE (mean absolute error), a number less than unity indicates that the model makes more precise forecasts compared to the random walk model. 2) DM p-val is a one-sided p-value of the Diebold-Mariano (Diebold and Mariano, 1995) test for equal forecast accuracy between full and finfric models. Its value below 0.05 implies that the precision of a model's forecasts is better than the alternative's at a 5% significance level. The results show that the full model's one-quarter ahead forecasts of GDP are statistically significantly more precise than those of the finfric model. 3) SVAR is estimated on three domestic variables: GDP, CPI and nominal interest rate, and is of the same structure and with similar priors as the foreign SVAR. 4) Note that this is not a true out-of-sample forecasting performance since the models have been estimated on the whole sample period 1995Q1-2012Q4. 5) 'finfric' - benchmark financial frictions model, 'full' - full model with unemployment, 'SVAR' - Bayesian SVAR model serving as another benchmark.

Latent labor market variables This subsection ends with a few smoothed latent labor market variables, shown in Figure 4. The smoothed probability of filling a vacancy within a quarter (Figure 4, upper left panel) overshoots in a few occasions, but otherwise looks reasonable.

The Cobb-Douglas labor matching technology employed by CTW is a popular choice in the literature but it does not ensure that the matching probability is proper, i.e. bounded in the interval $[0, 1]$. Therefore, den Haan, Ramey and Watson (2000, henceforth dHRW) came up with an alternative matching technology which ensures a proper matching probability. For robustness, Figure 4 shows the results for both matching functions, with dHRW matching function being

$$m_t = \frac{(1 - L_t)v_t}{\left((1 - L_t)^l + v_t^l\right)^{\frac{1}{l}}} \tag{3.2}$$

with a particular calibrated value $l = 1.36$ for Latvian data[15]. If not clearly stated otherwise, all the other results are produced using the Cobb-Douglas specification.

The steady-state value of quarterly job finding rate is 0.28 but its smoothed value (Figure 4, upper right panel) overshoots significantly during the boom period right before

[15] dHRW use $l = 1.27$ for US data.

the 2008-recession. This is because the smoothed level of unemployment rate[16] is underestimated during that period (Figure 4, bottom panel).

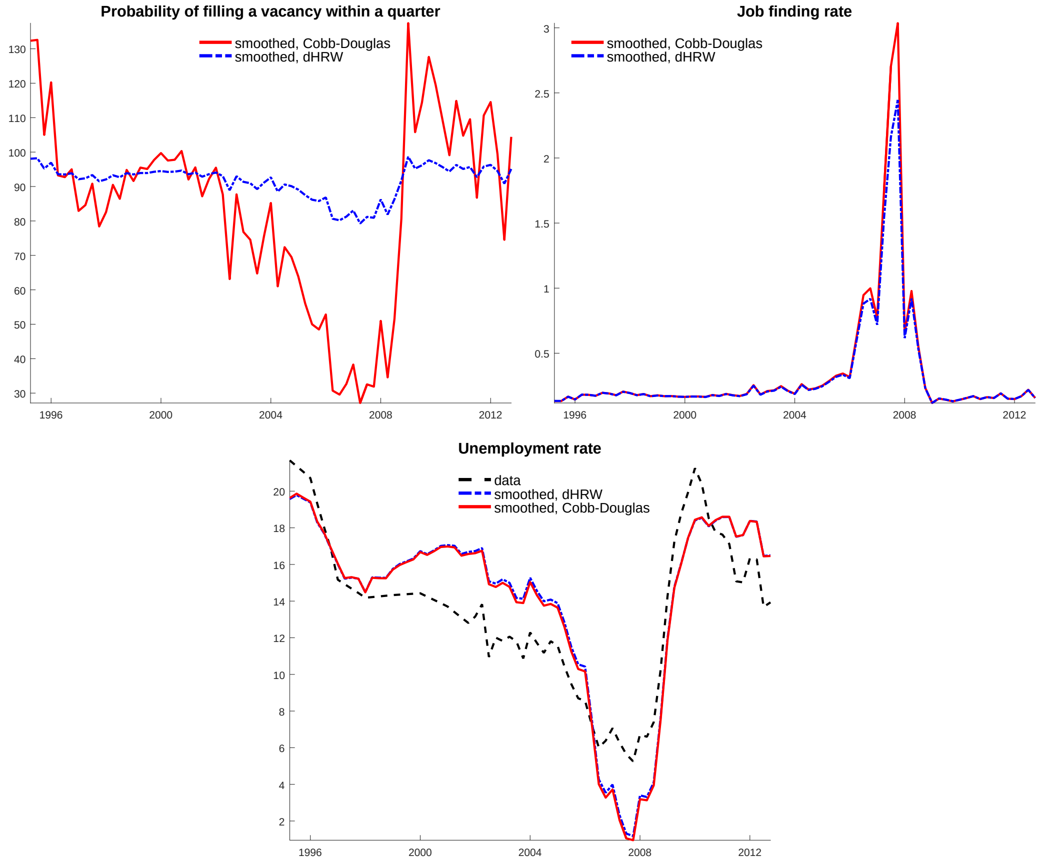

Figure 4: Latent labor market variables, Cobb-Douglas versus den Haan, Ramey and Watson (2000) matching function.

Note: 1) Probability of filling a vacancy within a quarter is defined as the ratio of total job matches over total job vacancies times 100, $Q = 100\frac{m_t}{v_t}$. 2) Job finding rate within a quarter is defined as the ratio of total job matches over total unemployment, $f_t = \frac{m_t}{1-L_t}$. 3) Unemployment rate is 'latent', as the model is fed with first differences of unemployment rate.

3.2 Vacancies observed

This subsection adds quarterly growth rate of vacancies as an observable and estimates the matching function together with an AR(1) shock to matching efficiency. It turns out that this brings a few results deemed implausible and thus highlight a possible model misspecification.

First, the posterior mean of unemployment share in the matching function decreases to 0.37 from a prior 0.5 (Table 1), thus outside the range considered to be sound, [0.5,0.7] (Shimer, 2005).

Second, during the boom period 2004-2007, one-period ahead forecasts of vacancies display dynamics opposite to the data, i.e. the forecasted vacancies tend to decrease -

[16]The model is fed with quarterly growth rates of unemployment rate.

generating positive correlation between vacancies and unemployment -, whereas the data increase during this period (Figure 5, left panel). This result is due to the attempt of the model to fit the volatility of vacancies, which now is slightly closer to the data yet still 2.2 times higher (Table 2). However, the better fit of vacancy volatility comes at the cost of a worse fit of the correlation between (first differenced) unemployment and vacancies, which decreases from -0.40 to -0.30 (Table 2). The one-quarter ahead forecasts of vacancies are to be compared to those in the previous subsection, where they, though having too high short-term volatility, clash less with the data in business-cycle frequencies (Figure 3, bottom right panel).

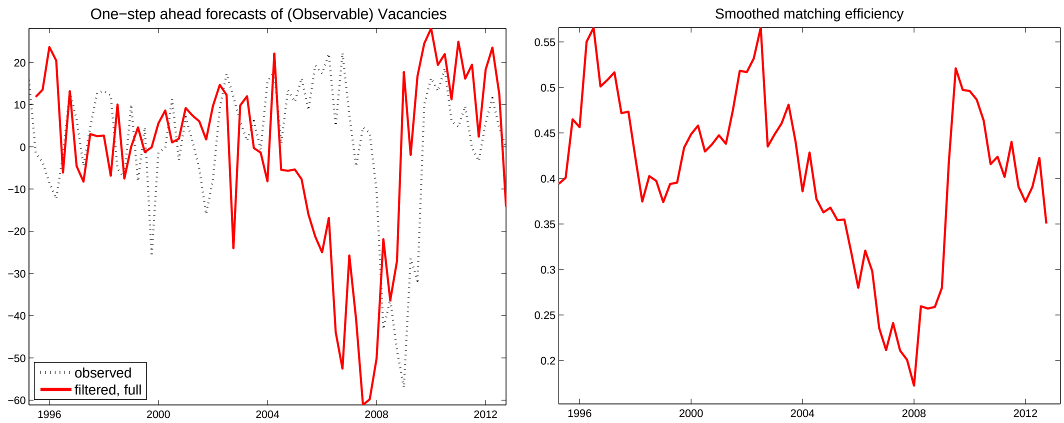

Figure 5: Implausible results when a shock to matching efficiency is allowed for.

Third, the smoothed AR(1) process of matching efficiency is counter-intuitively counter-cyclical - matching efficiency, which commonly is referred to be negatively related to structural unemployment, decreases during the boom 2004-2007, and increases thereafter during the recession (Figure 5, right panel).

4 Summary and conclusions

This paper examines, in an estimated, full-fledged New Keynesian DSGE model with Nash wage bargaining, sticky wage and high value of leisure akin to Christiano, Trabandt and Walentin (2011), whether search-and-matching frictions in labor market can explain aggregate labor market dynamics in Latvia. The paper adds to the literature by studying the ability of a richly specified New Keynesian model with search-and-matching frictions to fit the key moments of unemployment and vacancies, particularly for a non-US country.

The results are as follows. If vacancies are not observed, the model can, to a reasonable degree, generate realistic variance and dynamics of unemployment, and correlation between unemployment and (latent) vacancies, but at the expense of too volatile vacancies. As a by-product, one-quarter ahead forecasts of hours worked and GDP exhibit less excess volatility and thus are more precise, compared to a model without search-and-matching frictions.

However, if both unemployment and vacancies are observed and a shock to the matching efficiency is allowed for, then the cyclical behavior of forecasted vacancies - equivalently, the correlation between unemployment and vacancies - tends to counter the data (to the benefit of better fit of vacancies variance), and the smoothed matching efficiency counterintuitively decreases during the boom and increases during the recession.

The results tend to be different than for the US (e.g. by CET), calling for more studies across economies. As the next step, it is instructive to test AOB model to Latvian data.

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Appendix A Calibration and estimation details

For space considerations, the information regarding the first specification (with calibrated matching function) is shown. The results for the model without search-and-matching frictions are taken from Buss (2014).

A.1 Calibration

The calibrated values are displayed in Tables 5 and 6. These are the parameters that are typically calibrated in the literature and are related to "great ratios" and other observable quantities related to steady state values. The values of the parameters are selected such that they would be specific to the data at hand. Sample averages are used when available. I am using the calibrated values of Buss (2014) for the parameters common between the full and the benchmark financial frictions models.

Parameter	Value	Description
		Core block
α	0.400	Capital share in production
β	0.995	Discount factor
ω_c	0.450	Import share in consumption goods
ω_i	0.650	Import share in investment goods
ω_x	0.300	Import share in export goods
$\tilde{\phi}_a$	0.010	Elasticity of country risk to net asset position
η_g	0.202	Government spending share of GDP
τ_k	0.100	Capital tax rate
τ_w	0.330	Payroll tax rate
τ_c	0.180	Consumption tax rate
τ_y	0.300	Labor income tax rate
τ_b	0.000	Bond tax rate
μ_z	1.005	Steady state growth rate of neutral technology
μ_ψ	1	Steady state growth rate of investment technology
$\bar{\pi}$	1.005	Steady state gross inflation target
$\lambda_{d;m,c;m,i}$	1.300	Price markup for domestic, imp for cons, imp for inv
$\lambda_{x;m,x}$	1.200	Price markup for exports, imp for exp
ϑ_w	1.000	Wage indexation to real growth trend
$\varkappa^j$	$1-\kappa^j$	Indexation to inflation target for $j = d; x; m, c; m, i; m, x; w$
$\breve{\pi}$	1.005	Third indexing base
$\tilde{\phi}_S$	0	Country risk adjustment coefficient
		Financial frictions block
$F(\bar{\omega})$	0.020	Steady state bankruptcy rate
$100W_e/y$	0.100	Transfers to entrepreneurs
		Labor market frictions block
L	0.863	Steady state fraction of employment (1 - unemployment rate)
N	4.000	Number of agency cohorts/length of wage contracts
φ	2	Curvature of hiring costs
ρ	0.970	Exogenous survival rate of a match
σ	0.500	Unemployment share in matching technology
σ_m	0.400	Level parameter in matching function
ι	1.000	Employment adj. costs dependence on tightness, V/U

Table 5: Calibrated parameters.

The discount factor, β, and the tax rate on bonds, τ_b, are set to match roughly the sample average real interest rate for the euro area. The capital share, α, is set to 0.4. Import shares are set to reasonable values by consulting to the input-output tables and fellow economists - 45%, 65% and 30% for import share in consumption, investment and exports, respectively.[17] The government spending share in the gross

[17]The import share in exports has been reduced to 30% (from 55% in Buss, 2014) due to Stehrer

domestic product (henceforth, GDP) is set to match the sample average, i.e. 20.2%. The steady state growth rates of neutral technology and inflation are set to two percent annually, and correspond to the euro area. The steady state growth rate of investment-specific technology is set to zero. The steady state quarterly bankruptcy rate is calibrated to two percent, up from one percent in the CTW model for the Swedish data. The values of the price markups are set to the typical values found in the literature, i.e., to 1.2 for exports, and imports for exports, and 1.3 for the domestic, imports for consumption and imports for investment.

There is full indexation of wages to the steady state real growth, $\vartheta_w = 1$. The other indexation parameters are set to get the full indexation and thereby avoid steady state price and wage dispersion, following CTW. Tax rates are calibrated such that those would represent implicit or effective rates. Three of these are calibrated using Eurostat data[18]: tax rate on capital income is set to 0.1, the value-added tax on consumption, τ^c, and the personal income tax rate that applies to labor, τ^y, are set to $\tau^c = 0.18$ and $\tau^y = 0.3$. Payroll tax rate is set to $\tau^w = 0.33$, down from the official 0.35 (0.24 by employer and 0.11 by employee). The elasticity of country risk to net asset position, $\tilde{\phi}_a$ is set to a small positive number and, in that region, its purpose is to induce a unique steady state for the net foreign asset position. Transfers to entrepreneurs parameter W_e/y is kept the same as in CTW. The country risk adjustment coefficient in the uncovered interest parity condition is set to zero in order to impose the nominal interest rate peg.

For the labor block, the steady-state unemployment rate is set to the sample average. The length of wage contract N is set to annual negotiation frequency, as in CTW. The curvature of hiring costs is set to quadratic. The exogenous survival rate of the match is set to 0.97, similar to that in CTW, and to yield a reasonable steady-state job finding rate of 0.28. The matching function parameter σ is set so that the number of unemployed and vacancies have equal factor shares in the production of matches[19]. The level parameter in matching, σ_m, is calibrated to be 0.4, down from 0.57 in CTW, reflecting the fact that the natural level of unemployment is higher in Latvia than in Sweden. Its particular value is preferred by the model fit in terms of MDD. As in CTW, I assume hiring costs not search costs, thus $\iota = 1$. Endogenous breakups are determined using employer surplus only.[20]

Three observable ratios are chosen to be exactly matched throughout the estimation, and therefore three corresponding parameters are recalibrated for each parameter draw: the steady state real exchange rate, $\tilde{\varphi}$, to match the export share of GDP in the data, the scaling parameter for disutility of labor, A_L, to fix the fraction of their time that individuals spend working[21], and the entrepreneurial survival rate, γ, is set to match the

(2013) who suggest, from the value-added perspective, that share is about 30%. Such a change reduces somewhat the log marginal data density (by about one point) and the importance of the shock to imports for exports markup.

[18]Source: `http://epp.eurostat.ec.europa.eu/cache/ITY_PUBLIC/2-29042013-CP/EN/2-29042013-CP-EN.PDF`, accessed in September 6, 2013

[19]Shimer (2005) estimates σ to be 0.72 for the US data. The so called Hosios condition relates this parameter one-to-one to the worker's bargaining power (e.g. Amaral and Tasci (2012)).

[20]The choice is backed by better model fit to the data. It is also the choice of CTW who argue that including worker surplus in the separation criteria would introduce a tendency for job separations to decrease at the beginning of recessions as the value to the worker of holding on her job then increases, but this tendency appears counterfactual.

[21]This calibrated fraction of time spent working differs between the benchmark and the full models -

net worth to assets ratio[22]. Comparing across the models, the implied posterior mean of the scaling parameter for disutility of labor is considerably higher for the model with unemployment compared to the benchmark model.

In the earlier steps of calibration, the depreciation rate of capital, δ, was also set to match the ratio of investment over output, but the realized value of depreciation rate turned out to be rather high (unless the capital share in production, α, was substantially increased but that yielded excessively high capital to output ratio) and sensitive to the initial values, therefore it was decided to fix the quarterly depreciation rate to a more reasonable value of three percent.

	Parameter description	Posterior mean		Moment	Moment value
		finfric	full		
$\tilde{\varphi}$	Real exchange rate	2.04	0.88	$SP^x X/(PY)$	0.462
A_L	Scaling of disutility of work	37.81	570011.41	$L\varsigma$	0.240
γ	Entrepreneurial survival rate	0.96	0.96	$n/(p_{k'}k)$	0.600

Table 6: Matched moments and corresponding parameters.

Note: The quarterly depreciation rate of capital is fixed at three percent. 'finfric' - benchmark financial frictions model, 'full' - full model with unemployment.

A.2 Shocks and measurement errors

In total, there are 21 exogenous stochastic variables in the full model: four technology shocks - stationary neutral technology, ϵ, stationary marginal efficiency of investment, Υ, unit-root neutral technology, μ_z, and unit-root investment specific technology, μ_Ψ, - a shock to consumption preferences, ζ^c, and to disutility of labor supply, ζ^h, a shock to government spending, g, and a country risk premium shock that affects the relative riskiness of foreign assets compared to domestic assets, $\tilde{\phi}$. There are five markup shocks, one for each type of intermediate good, τ^d, τ^x, $\tau^{m,c}$, $\tau^{m,i}$, $\tau^{m,x}$ (d - domestic, x - exports, m, c - imports for consumption, m, i - imports for investment, m, x - imports for exports). The financial frictions block has two shocks - one to idiosyncratic uncertainty, σ, and one to entrepreneurial wealth, γ. There are also shocks to each of the foreign observed variables - foreign GDP, y^*, foreign inflation, π^*, and foreign nominal interest rate, R^*.

The employment frictions block adds three shocks - a shock to the bargaining power of workers, η, a shock to the matching productivity, σ_m, and a shock to the productivity dispersion among workers, affecting the endogenous job separations, σ_a.

The stochastic structure of the exogenous variables are the following: 11 of these evolve according to AR(1) processes:

whereas it is 0.27 for the benchmark model, it is lowered to 0.24 for the full model due to the existence of unemployment in the latter. Both values are somewhat arbitrary but checked against the model fit with respect to their neighboring values.

[22]The net worth to assets ratio for Latvia, if the definition of CTW is taken, yields about 0.15. However, the model fit favors a much larger number, 0.6, which is used in the final calibration. The latter number might be rationalized if the net worth was measured not only by the share price index but if it included also the real estate value.

$$\epsilon_t, \Upsilon_t, \zeta_t^c, \zeta_t^h, g_t, \tilde{\phi}_t, \sigma_t, \gamma_t, \eta_t, \sigma_{m,t}, \sigma_{a,t}$$

Five shock processes are i.i.d.:

$$\tau_t^d, \tau_t^x, \tau_t^{m,c}, \tau_t^{m,i}, \tau_t^{m,x}$$

and five shock processes are assumed to follow an SVAR(1):

$$y_t^*, \pi_t^*, R_t^*, \mu_{z,t}, \mu_{\Psi,t}.$$

Four shocks are suspended in the estimation: the shock to unit-root investment specific technology, $\mu_{\Psi,t}$, the idiosyncratic entrepreneurial risk shock, σ_t, the shock to bargaining power, η_t, and the shock to the standard deviation of idiosyncratic productivity of workers, $\sigma_{a,t}$. The first one should correspond to the foreign block but its identification is dubious in the particular SVAR model. The second has been found to have limited importance in CTW. Also, CTW argue that shocks to η_t seem superfluous as we already have the standard labor supply shock - the labor preference shock, ζ_t^h. In the model version where vacancies are not observed, the shock to matching technology, $\sigma_{m,t}$, is also suspended.

There are measurement errors except for the domestic interest rate and the foreign variables. The variance of the measurement errors is calibrated to correspond to 10% of the variance of each data series.

A.3 Priors

There are 24 structural parameters, eight AR(1) coefficients, 16 SVAR parameters for the foreign economy, and 16 shock standard deviations estimated with Bayesian techniques within Matlab/Dynare environment (Adjemian et al, 2011). The priors are displayed in Tables 7 to 10. The priors common to the benchmark financial frictions model are taken from Buss (2014).

Regarding the three new parameters in the labor block, for hiring costs as a fraction of GDP, *hshare*, I use a prior with mean of 0.3% up from 0.1% in CTW in order to move it closer to the posterior. The prior mean of *bshare*, the ratio of the flow value of utility provided to the household of a worker of being unemployed to the flow value of utility of a worker being employed is 0.75, as in CTW. The prior mean of the endogenous employer-employee match separation rate, $F, \%$ is 0.25%, i.e. roughly 7.7% of the total job separation rate, similar to CTW.

	Parameter description	Distr.	Prior Mean	Prior st.d.	Posterior Mean	Posterior st.d.	HPD int. 10%	HPD int. 90%
ρ_{μ_z}	Persistence, unit-root tech.	β	0.50	0.075	0.590	0.063	0.487	0.696
a_{11}	Foreign SVAR parameter	N	0.90	0.05	0.913	0.034	0.852	0.977
a_{22}	Foreign SVAR parameter	N	0.50	0.05	0.521	0.055	0.438	0.605
a_{33}	Foreign SVAR parameter	N	0.90	0.05	0.954	0.023	0.919	0.989
a_{12}	Foreign SVAR parameter	N	-0.10	0.10	-0.165	0.091	-0.314	-0.016
a_{13}	Foreign SVAR parameter	N	-0.10	0.10	-0.045	0.054	-0.124	0.037
a_{21}	Foreign SVAR parameter	N	0.10	0.10	0.181	0.043	0.097	0.260
a_{23}	Foreign SVAR parameter	N	-0.10	0.10	-0.090	0.055	-0.183	-0.008
a_{24}	Foreign SVAR parameter	N	0.05	0.10	0.078	0.041	0.009	0.146
a_{31}	Foreign SVAR parameter	N	0.05	0.10	0.080	0.029	0.032	0.131
a_{32}	Foreign SVAR parameter	N	-0.10	0.10	-0.095	0.058	-0.198	0.002
a_{34}	Foreign SVAR parameter	N	0.10	0.10	0.108	0.026	0.068	0.149
c_{21}	Foreign SVAR parameter	N	0.05	0.05	0.021	0.040	-0.048	0.088
c_{31}	Foreign SVAR parameter	N	0.10	0.05	0.145	0.031	0.094	0.196
c_{32}	Foreign SVAR parameter	N	0.40	0.05	0.374	0.053	0.286	0.459
c_{24}	Foreign SVAR parameter	N	0.05	0.05	0.065	0.046	-0.003	0.135
c_{34}	Foreign SVAR parameter	N	0.05	0.05	0.048	0.034	-0.002	0.101

Table 7: Estimated foreign SVAR parameters.

Note: Based on a single Metropolis-Hastings chain with 100 000 draws after a burn-in period of 900 000 draws.

	Description	Distr.	Prior Mean	Prior st.d.	Posterior Mean	Posterior st.d.	HPD int. 10%	HPD int. 90%
$100\sigma_{\mu_z}$	Unit root technology	Inv-Γ	0.25	inf	0.328	0.052	0.248	0.406
$100\sigma_{y^*}$	Foreign GDP	Inv-Γ	0.50	inf	0.317	0.055	0.219	0.415
$1000\sigma_{\pi^*}$	Foreign inflation	Inv-Γ	0.50	inf	0.593	0.118	0.394	0.805
$100\sigma_{R^*}$	Foreign interest rate	Inv-Γ	0.075	inf	0.067	0.008	0.054	0.079

Table 8: Estimated standard deviations of SVAR shocks.

Note: Based on a single Metropolis-Hastings chain with 100 000 draws after a burn-in period of 900 000 draws.

	Parameter description	Distr.	Prior Mean	Prior st.d.	Posterior Mean finfric	Posterior Mean full	Posterior st.d. finfric	Posterior st.d. full	HPD int. 10% full	HPD int. 90% full
ξ_d	Calvo, domestic	β	0.75	0.075	0.804	0.810	0.023	0.020	0.766	0.851
ξ_x	Calvo, exports	β	0.75	0.075	0.860	0.883	0.031	0.023	0.831	0.930
ξ_{mc}	Calvo, imports for consumpt.	β	0.75	0.075	0.779	0.802	0.049	0.053	0.716	0.895
ξ_{mi}	Calvo, imports for investment	β	0.65	0.075	0.408	0.385	0.042	0.047	0.275	0.493
ξ_{mx}	Calvo, imports for exports	β	0.65	0.10	0.589	0.592	0.091	0.040	0.425	0.770
κ_d	Indexation, domestic	β	0.40	0.15	0.162	0.306	0.075	0.101	0.114	0.506
κ_x	Indexation, exports	β	0.40	0.15	0.301	0.392	0.107	0.098	0.171	0.612
κ_{mc}	Indexation, imports for cons.	β	0.40	0.15	0.366	0.480	0.106	0.102	0.244	0.740
κ_{mi}	Indexation, imports for inv.	β	0.40	0.15	0.249	0.303	0.100	0.102	0.097	0.511
κ_{mx}	Indexation, imports for exp.	β	0.40	0.15	0.317	0.337	0.115	0.069	0.104	0.590
κ_w	Indexation, wages	β	0.40	0.15	0.241	0.240	0.079	0.083	0.083	0.416
ν^j	Working capital share	β	0.50	0.25	0.456	0.471	0.179	0.207	0.041	0.902
$0.1\sigma_L$	Inverse Frisch elasticity	Γ	0.30	0.15	0.287	1.004	0.106	0.113	0.653	1.386
b	Habit in consumption	β	0.65	0.15	0.898	0.878	0.030	0.030	0.806	0.947
$0.1S''$	Investment adjustm. costs	Γ	0.50	0.15	0.168	0.168	0.030	0.037	0.075	0.260
σ_a	Variable capital utilization	Γ	0.20	0.075	0.567	0.398	0.093	0.058	0.206	0.641
η_x	Elasticity of subst., exports	Γ_{tr}	1.50	0.25	1.535	1.574	0.143	0.176	1.178	1.991
η_c	Elasticity of subst., cons.	Γ_{tr}	1.50	0.25	1.333	1.318	0.164	0.111	1.010	1.651
η_i	Elasticity of subst., invest.	Γ_{tr}	1.50	0.25	1.1*	1.261		0.091	1.010	1.565
η_f	Elasticity of subst., foreign	Γ_{tr}	1.50	0.25	1.540	1.523	0.159	0.243	1.070	1.990
μ	Monitoring cost	β	0.30	0.075	0.273	0.261	0.040	0.033	0.182	0.341
$hshare, \%$	Hiring costs	Γ	0.30	0.075		0.380		0.062	0.247	0.518
$bshare$	Utility flow, unemployed	β	0.75	0.075		0.790		0.038	0.694	0.882
$F, \%$	Endogenous separation rate	β	0.25	0.05		0.358		0.026	0.298	0.416
ρ_ϵ	Persistence, stationary tech.	β	0.85	0.075	0.847	0.843	0.041	0.054	0.717	0.950
ρ_Υ	Persistence, MEI	β	0.85	0.075	0.588	0.585	0.106	0.073	0.364	0.989
ρ_{ζ^c}	Persist., consumption prefs	β	0.85	0.075	0.851	0.848	0.038	0.037	0.743	0.937
ρ_{ζ^h}	Persistence, labor prefs	β	0.85	0.075	0.817	0.960	0.048	0.019	0.925	0.993
$\rho_{\tilde{\phi}}$	Persist., country risk prem.	β	0.85	0.075	0.934	0.899	0.025	0.025	0.845	0.949
ρ_g	Persist., gov. spending	β	0.85	0.075	0.777	0.776	0.083	0.056	0.611	0.932
ρ_γ	Persistence, entrep. wealth	β	0.85	0.075	0.796	0.792	0.059	0.069	0.608	0.967

Table 9: Estimated parameters.

Note: Based on two Metropolis-Hastings chains each with 100 000 draws after a burn-in period of 400 000 draws. * - calibrated in order to avoid numerical issues. Note that truncated priors, denoted by Γ_{tr}, with no mass below 1.01 have been used for the elasticity parameters η_j, $j = \{x, c, i, f\}$. 'finfric' - benchmark financial frictions model, 'full' - full model with unemployment.

	Description	Distr.	Prior Mean	Prior st.d.	Posterior Mean finfric	Posterior Mean full	Posterior st.d. finfric	Posterior st.d. full	HPD int. 10% full	HPD int. 90% full
$10\sigma_{\epsilon}$	Stationary technology	Inv-Γ	0.15	inf	0.126	0.134	0.014	0.016	0.103	0.166
σ_{Υ}	Marginal efficiency of invest.	Inv-Γ	0.15	inf	0.157	0.160	0.027	0.036	0.060	0.252
σ_{ζ^c}	Consumption prefs	Inv-Γ	0.15	inf	0.236	0.211	0.056	0.046	0.117	0.328
σ_{ζ^h}	Labor prefs	Inv-Γ	0.50	inf	0.895	0.290	0.283	0.030	0.212	0.382
$100\sigma_{\tilde{\phi}}$	Country risk premium	Inv-Γ	0.50	inf	0.552	0.550	0.045	0.048	0.460	0.642
$10\sigma_g$	Government spending	Inv-Γ	0.50	inf	0.471	0.475	0.041	0.043	0.390	0.568
σ_{τ^d}	Markup, domestic	Inv-Γ	0.50	inf	0.373	0.433	0.089	0.087	0.251	0.651
σ_{τ^x}	Markup, exports	Inv-Γ	0.50	inf	0.992	1.519	0.391	0.605	0.598	2.786
$\sigma_{\tau^{m,c}}$	Markup, imports for cons.	Inv-Γ	0.50	inf	0.863	0.849	0.329	0.271	0.260	1.897
$\sigma_{\tau^{m,i}}$	Markup, imports for invest.	Inv-Γ	0.50	inf	0.433	0.412	0.078	0.080	0.262	0.593
$\sigma_{\tau^{m,x}}$	Markup, imports for exports	Inv-Γ	0.50	inf	1.383	2.298	0.643	0.438	0.673	4.853
$10\sigma_{\gamma}$	Entrepreneurial wealth	Inv-Γ	0.50	inf	0.295	0.275	0.042	0.043	0.202	0.360

Table 10: Estimated standard deviations of shocks.

Note: Based on two Metropolis-Hastings chains each with 100 000 draws after a burn-in period of 400 000 draws. 'finfric' - benchmark financial frictions model, 'full' - full model with unemployment.

A.4 Data

The model is estimated using data for Latvia ('domestic' part) and the euro area ('foreign' part). The sample period is 1995Q1 - 2012Q4. 19 observable time series are used to estimate the model specification without vacancies. Otherwise, these 19 variables plus first differenced vacancies are used for the second specification. The variables used in levels are: nominal interest rate, GDP deflator inflation, consumer price index (henceforth, CPI) inflation, investment price index inflation, foreign CPI inflation, foreign nominal interest rate and the interest rate spread. The rest of the variables are in terms of the first differences of logs, and these are: GDP, consumption, investment, exports, imports, government spending, real wage, real exchange rate, real stock prices, total hours worked, unemployment, and foreign GDP. The differenced variables are demeaned except for total hours worked and unemployment. The domestic inflation rates and the real exchange rate are demeaned as well. All real quantities are in per capita terms.

A.4.1 Posterior parameter values

The domestic and foreign blocks are estimated separately since Latvia's economy has minuscule effect on the euro area economy[23]. The estimation results for the foreign SVAR model are obtained using a single Metropolis-Hastings chain with 100 000 draws after a burn-in of 900 000 draws. For the domestic block, the estimation results are obtained using two Metropolis-Hastings chains, each with 100 000 draws after a burn-in of 400 000 draws. Prior-posterior plots are relegated to Appendix B.

The posterior parameter estimates for the foreign block are reported in Tables 7 - 8, and those specific to the domestic block - in Tables 9 - 10. For comparison, I also report the results for the domestic block of the benchmark financial frictions model.

[23]Latvia's share in the euro area is about 0.23% in terms of nominal GDP.

The major differences in the estimated mean parameter values between the models are the following. First, the homogeneous domestic good price indexation (to lagged inflation) parameter, κ_d, has moved closer to the prior mean from 0.16 (benchmark model) to 0.32 (full model), resulting in a more rigid estimated homogeneous good price inflation.

The inverse Frisch elasticity parameter σ_L (which captures the inverse elasticity of hours worked to the wage rate, given a constant marginal utility of wealth) has more than tripled from 2.9 to 9.7, above the 7.7 reported by CTW for Sweden. This means that the estimated (non-inverted) Frisch elasticity has decreased from a rather standard level, from the United States micro data perspective (Reichling and Whalen, 2012), of 0.34 to a rather low level of 0.1, indicating that employees vary their hours of work less in response to changes in their after-tax compensation.

The parameter governing the variable capital utilization, σ_a, has decreased from 0.57 to 0.36, signaling for more variation in capital utilization. The persistence parameter governing labor preferences has increased from 0.82 to 0.96, and is the only persistence parameter whose posterior mean is above 0.9.

Also, and similar to CTW, the estimated standard deviation of the labor preference shock, σ_{ζ^h} has decreased by a factor of three, compared to the benchmark model. Thus, the model without the search and matching frictions relies on large amounts of high frequency variation of this shock to explain the data.[24] See as well the graphical comparison of smoothed shocks in the Online Appendix.

Regarding the labor block, the posterior mean of utility flow parameter for the unemployed, $bshare$, is 0.80, above its prior mean (0.75), in line with Hagedorn and Manovskii (2008) that high value of leisure helps fit the volatility of unemployment.

The hiring costs as a fraction of GDP is estimated to be 0.39%, which is higher than the prior mean (0.3) and about the same as reported by CTW for Sweden.

The endogenous separation rate is estimated to be 0.36%, up from its prior 0.25%, implying that about 10.7% of job separations are endogenous[25], that is, cyclical, since the other part of the separations - the exogenous separation - is fixed and thus acyclical.[26] This endogenous separation rate is higher than the about 6% reported in CTW for Sweden for the period 1995Q1-2010Q3.

The bargaining power of workers, η, is solved for to yield a steady state unemployment rate matching the sample average. The value of η at the posterior mean is 0.65, which is higher than 0.29 reported by CTW for Sweden and slightly higher than 0.5 suggested by conventional wisdom (Mortensen and Nagypal, 2007). This result may be due to the 2005-boom period in Latvia, during which several sectors of the economy experienced shortage of workers.

[24] CTW interpretation of this difference is that the tight link between the desired real wage and total hours worked (through the marginal rate of substitution between leisure and consumption) implied by EHL labor market modeling does not hold in the data, even when this link is relaxed by assuming wage stickiness. This model instead implies efficient provision of labor on the intensive margin without any direct link to the (sticky, bargained) wage, and thereby allows for a high frequency disconnect between wages and hours worked. Fundamentally, as CTW notes, this model reflects that labor is not supplied on a spot market, but within long-term relationships.

[25] Calculated as: endogenous rate over the (exogenous plus (one minus exogenous) times endogenous rate).

[26] My unpublished results show that the share of the endogenous separation goes down to about 8% if more generous data measurement errors are allowed for, than the current 10% of data variance.

References

[1] Adjemian, Stephane, Bastani, Houtan, Juillard, Michel, Karame, Frederic, Mihoubi, Ferhat, Perendia, George, Pfeifer, Johannes, Ratto, Marco and Villemot, Sebastien, 2011. "Dynare: Reference Manual, Version 4" Dynare Working Papers, 1, CEPREMAP.

[2] Amaral, Pedro S. and Murat Tasci, 2012. "The cyclical behavior of equilibrium unemployment and vacancies across OECD countries", Working Paper 1236, Federal Reserve Bank of Cleveland.

[3] Reichling, Felix and Charles Whalen, 2012. "Review of estimates of the Frisch elasticity of labor supply", Working Paper 2012-13, Congressional Budget Office.

[4] Stehrer, Robert, 2013. "Accounting Relations in Bilateral Value Added Trade", wiiw Working Papers 101, The Vienna Institute for International Economic Studies, wiiw.

Appendix B Online appendix on computational details - not for publication

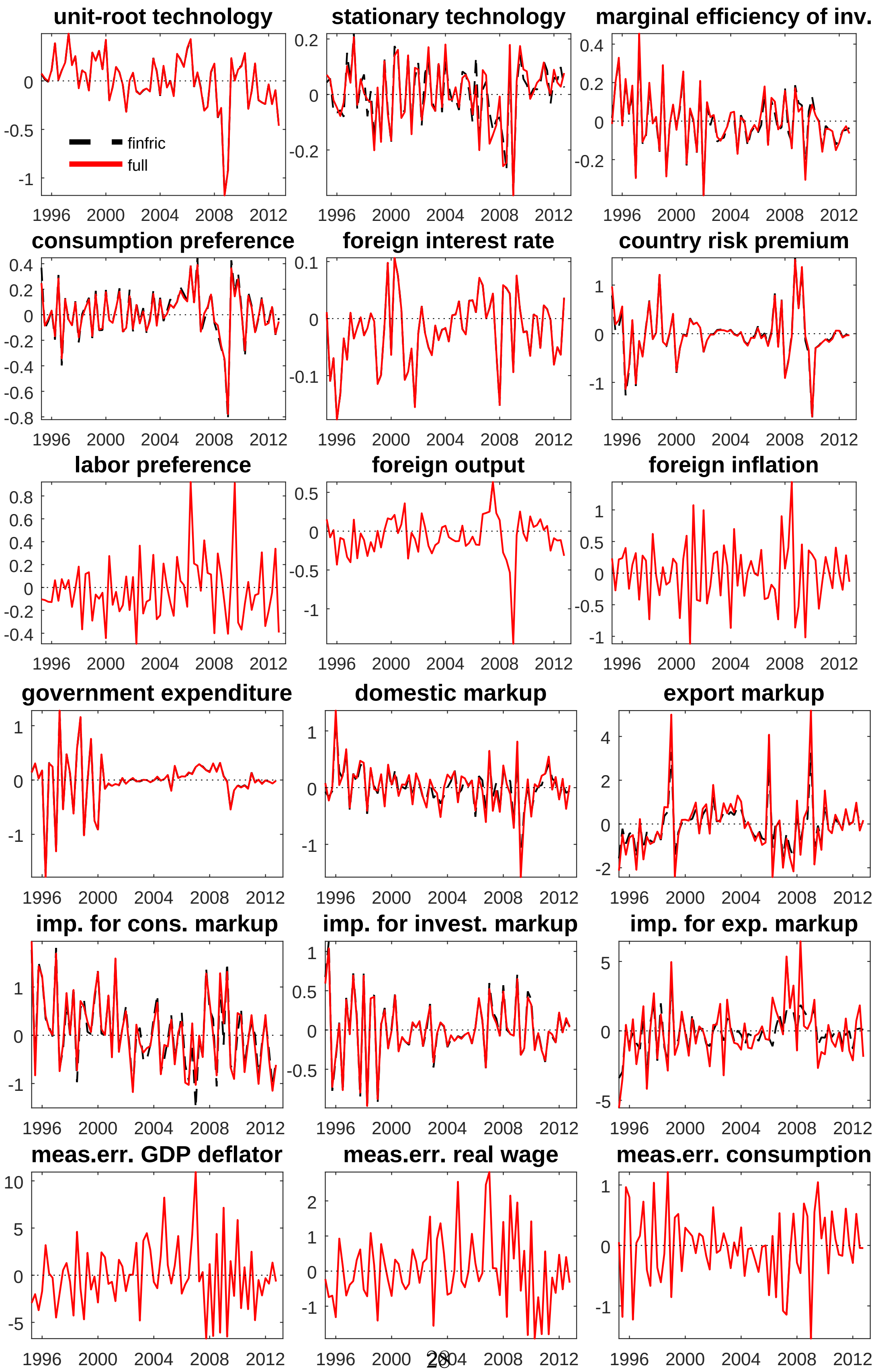

Figure 6: Smoothed shock processes, measurement errors and the innovation to the interest rate rule, ϵ_R.

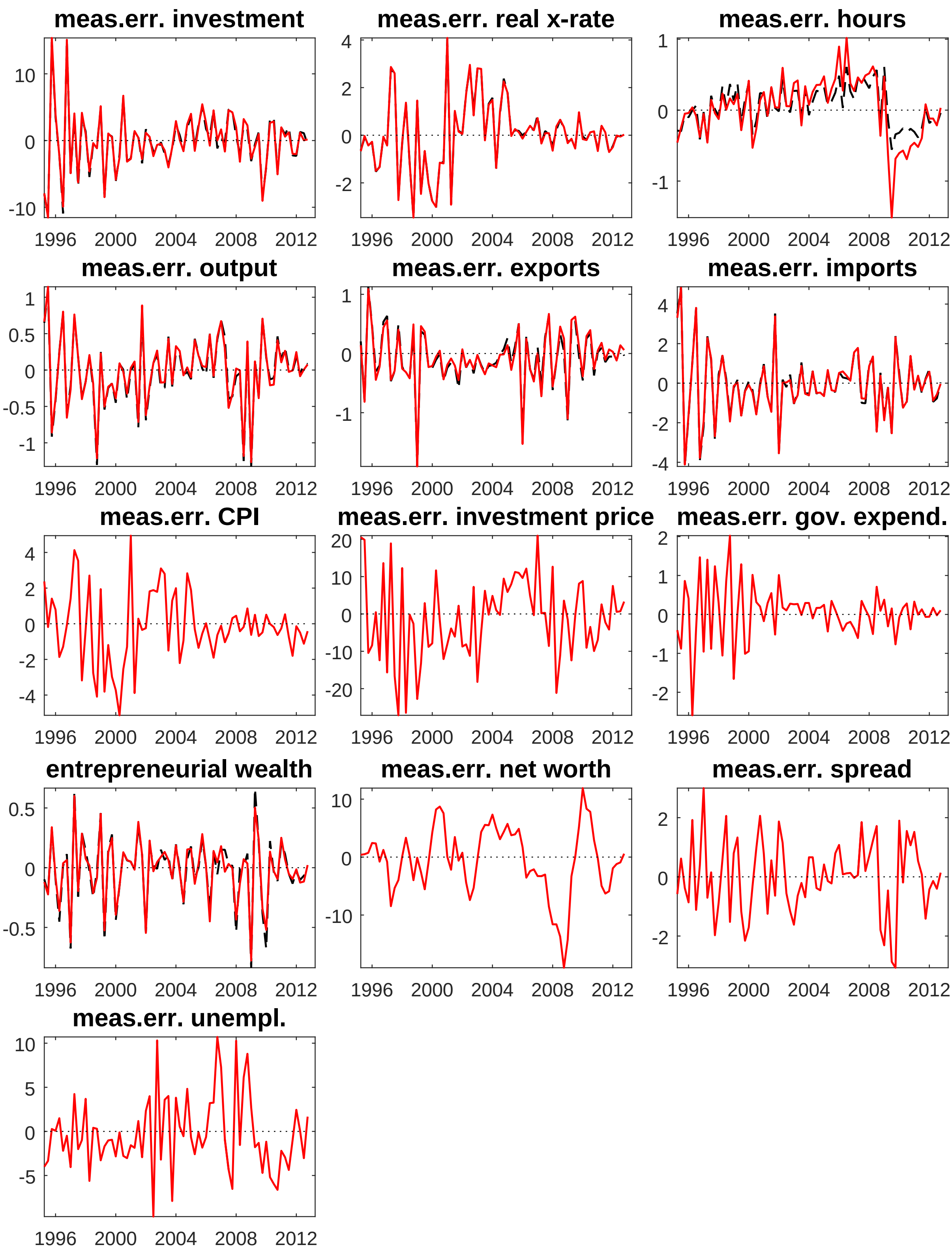

Figure 7: Smoothed shock processes, measurement errors and the innovation to the interest rate rule, ϵ_R. (Continued)

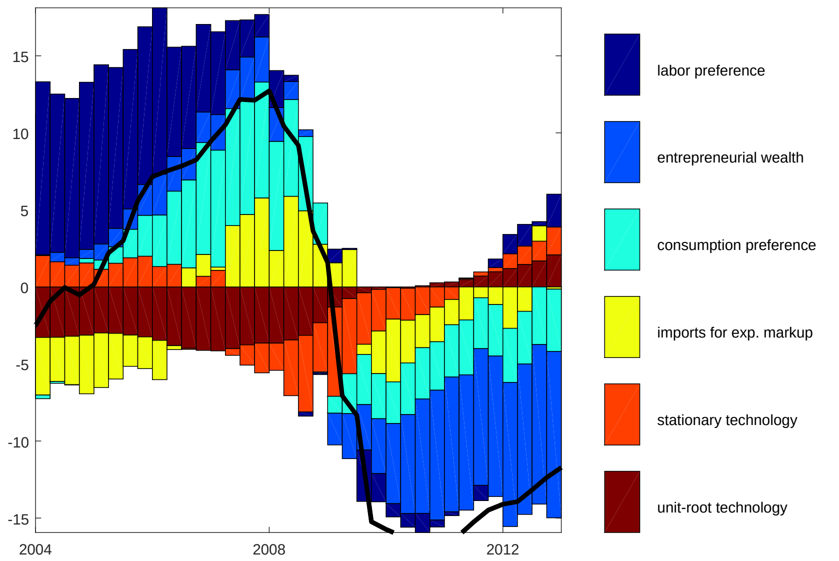

Figure 8: Decomposition of GDP (levels), 2004Q1-2012Q4.

Note: Full model. Only the six shocks with the greatest influence shown.

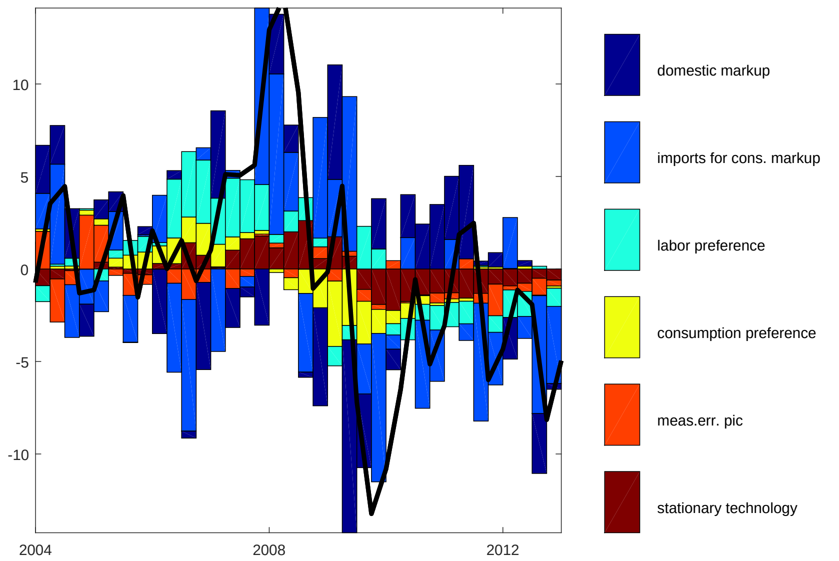

Figure 9: Decomposition of CPI (annualized quarterly growth rates), 2004Q1-2012Q4.

Note: Full model. Only the six shocks with the greatest influence shown.

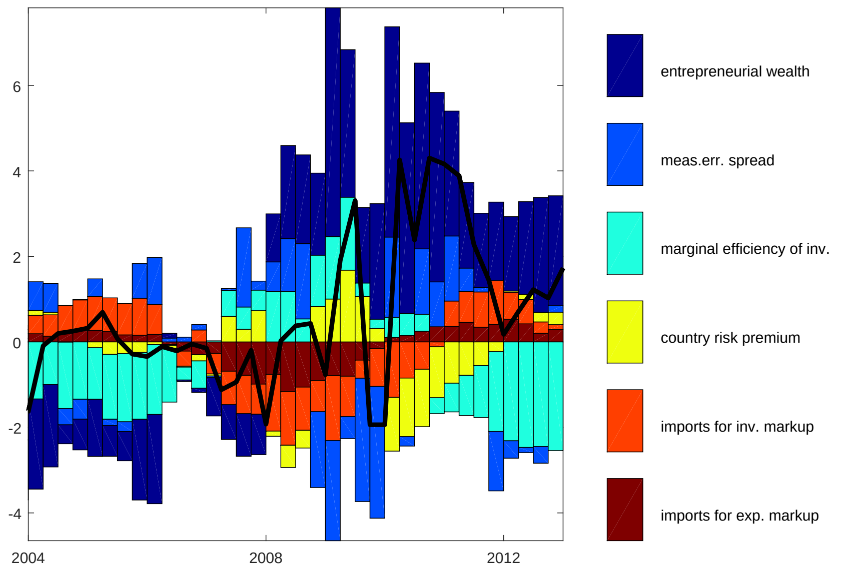

Figure 10: Decomposition of interest rate spread, $Z_{t+1} - R_t$, 2004Q1-2012Q4.

Note: Full model. Only the six shocks with the greatest influence shown.

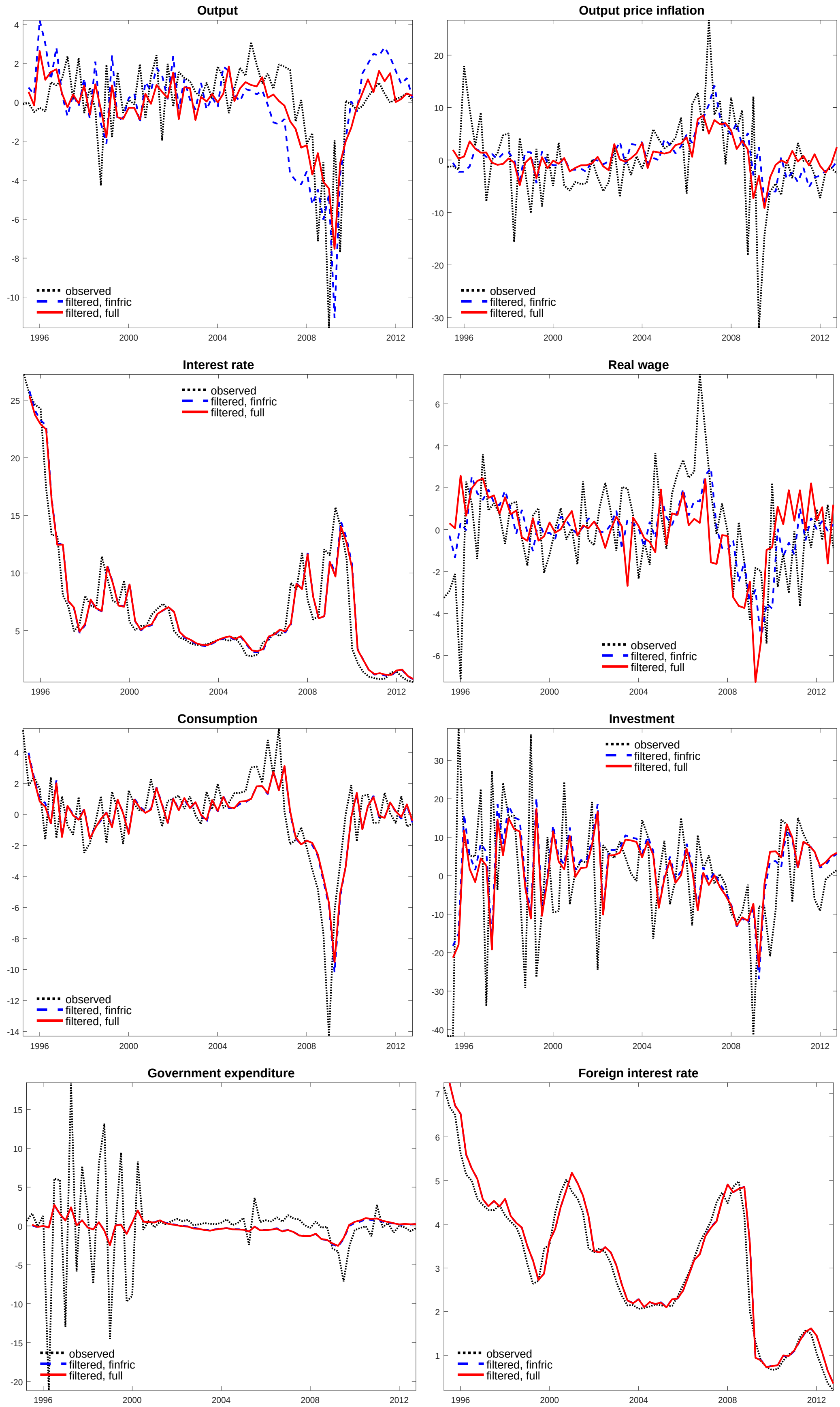

Figure 11: One-step ahead forecasts

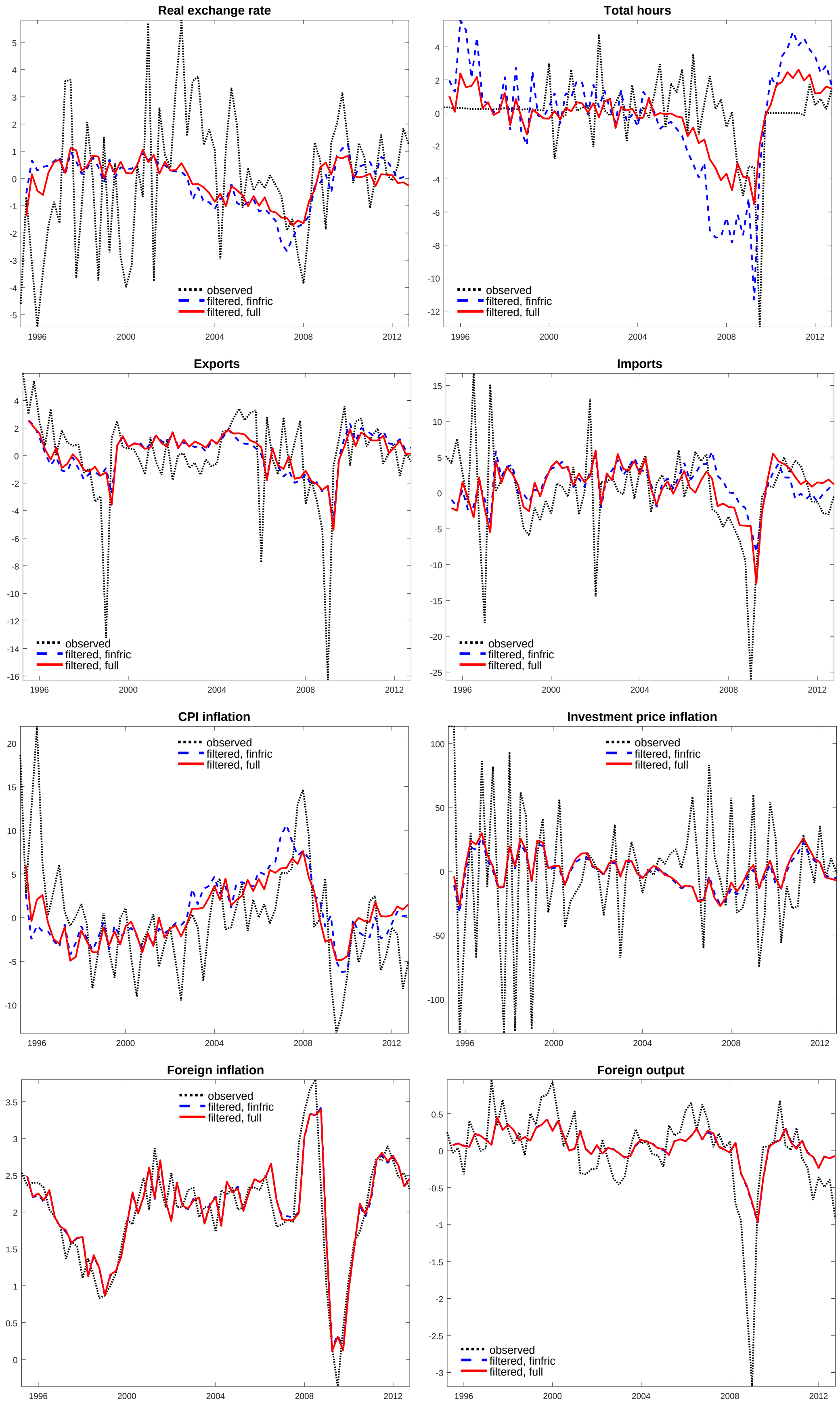

Figure 12: One-step ahead forecasts (continued)

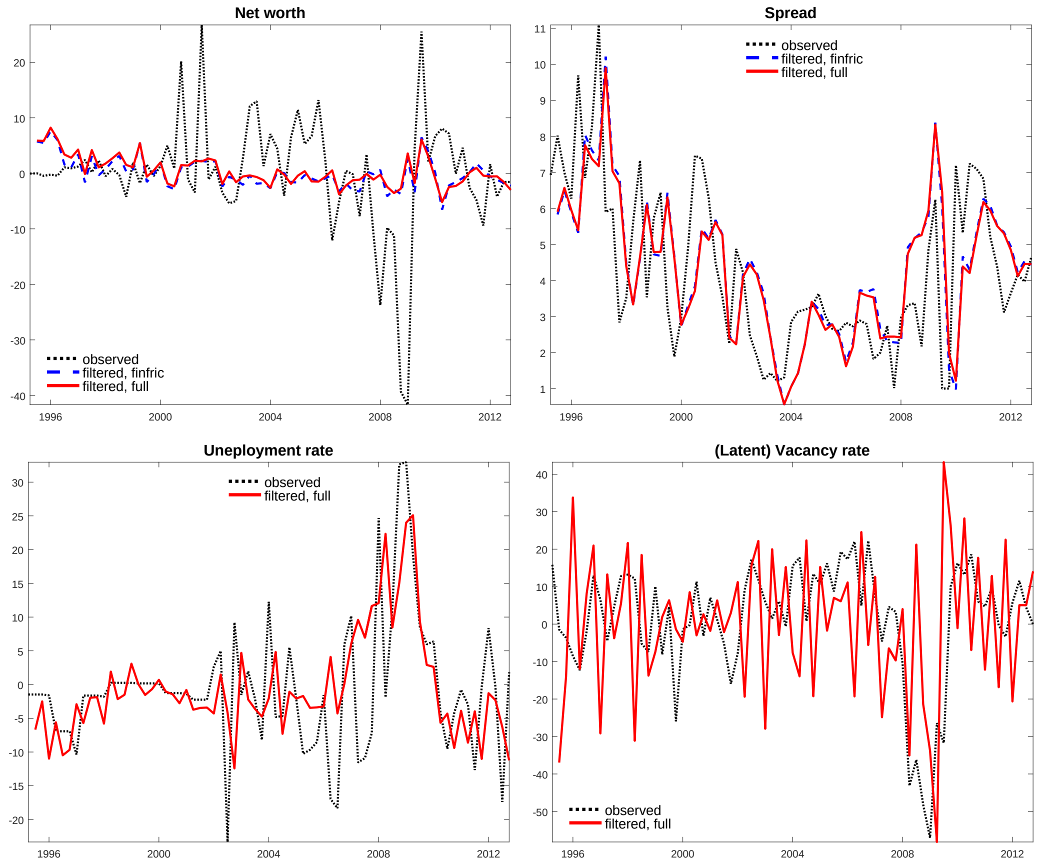

Figure 13: One-step ahead forecasts (continued)

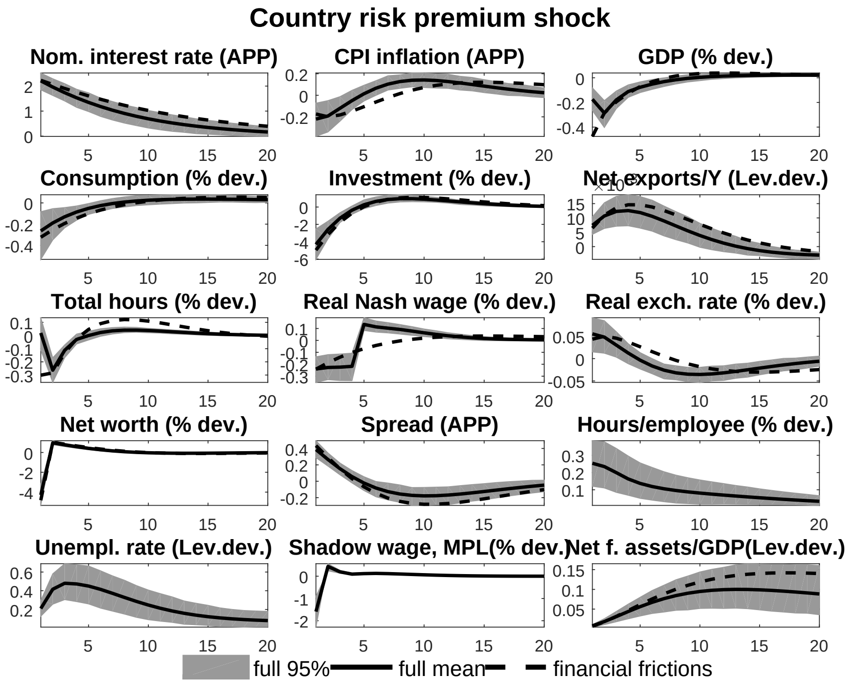

Figure 14: Impulse responses to the country risk premium shock, $\tilde{\phi}_t$. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

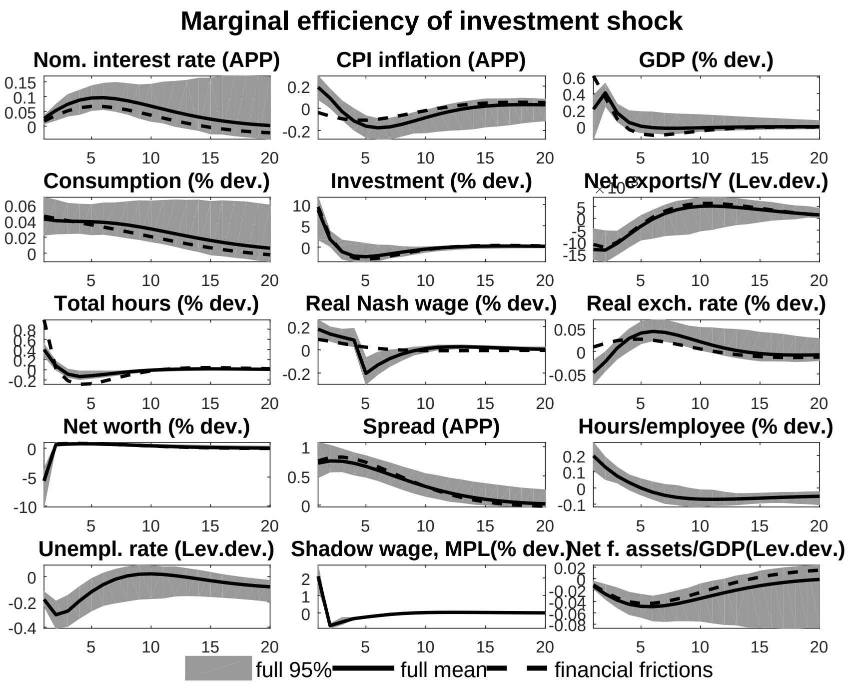

Figure 15: Impulse responses to the marginal efficiency of investment shock, Υ_t.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

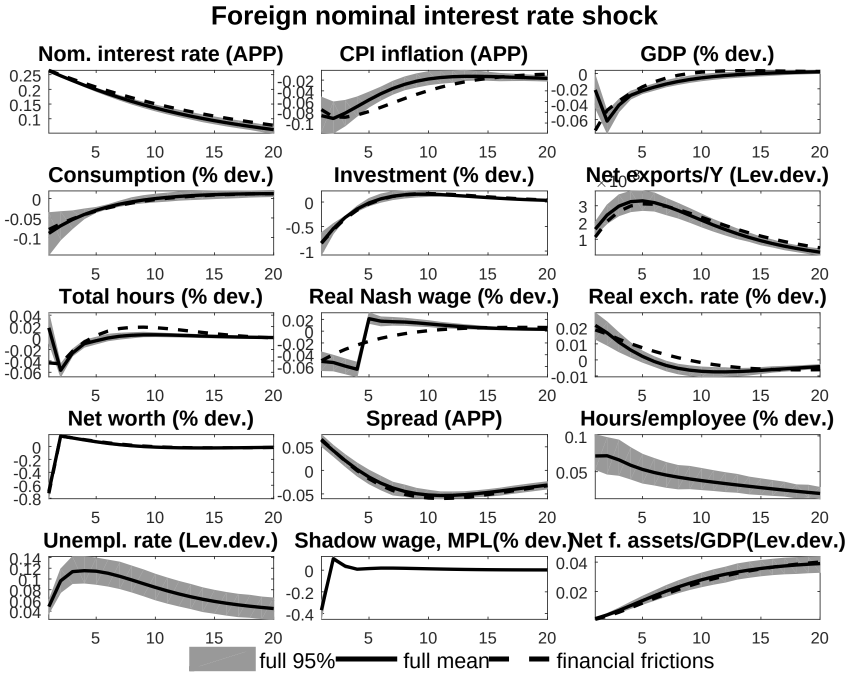

Figure 16: Impulse responses to the foreign nominal interest rate shock, $\epsilon_{R^*,t}$. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

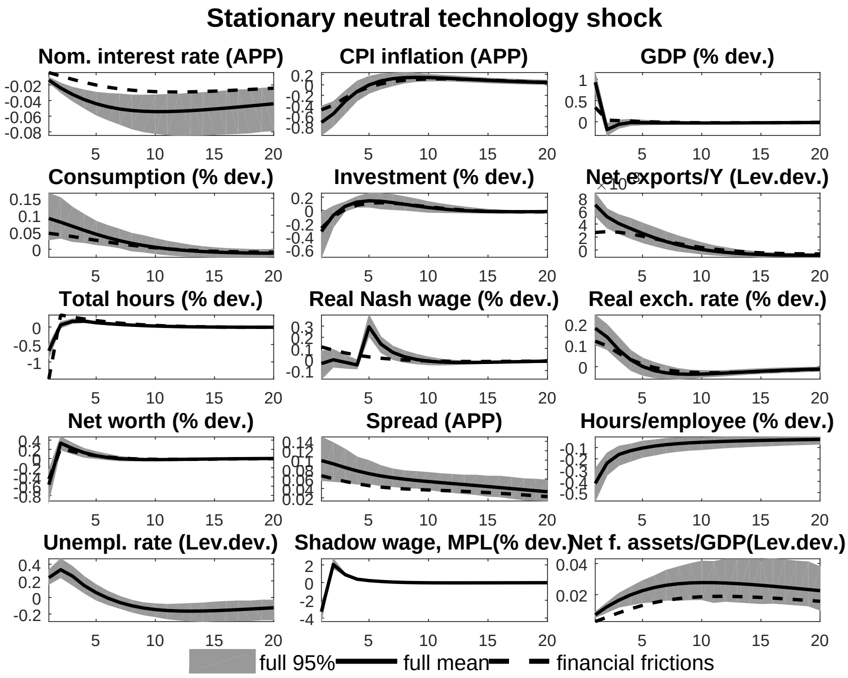

Figure 17: Impulse responses to the stationary neutral technology shock, ϵ_t.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

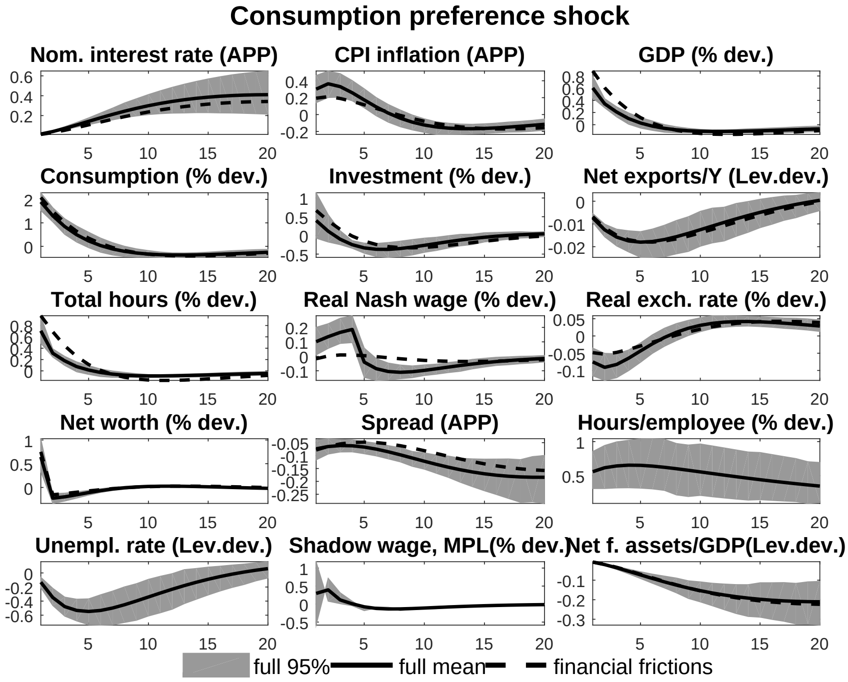

Figure 18: Impulse responses to the consumption preference shock, ζ_t^c. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

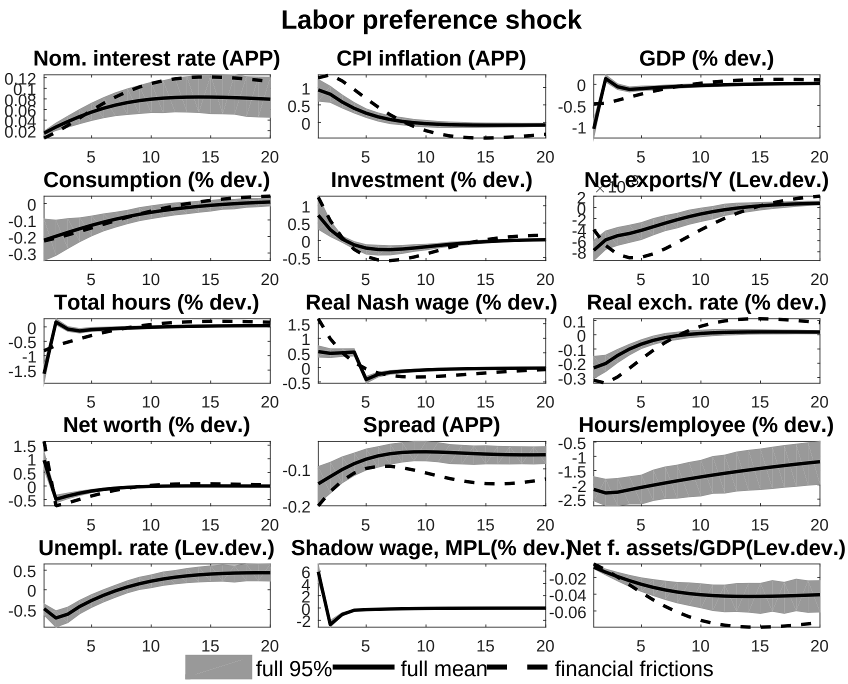

Figure 19: Impulse responses to the labor preference shock, ζ_t^h. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

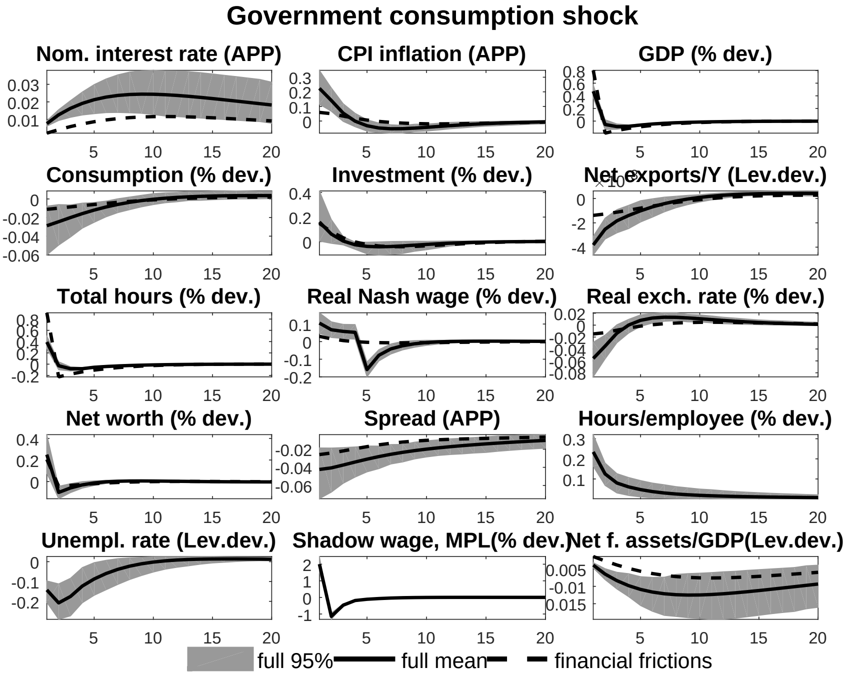

Figure 20: Impulse responses to the government consumption shock, g_t. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

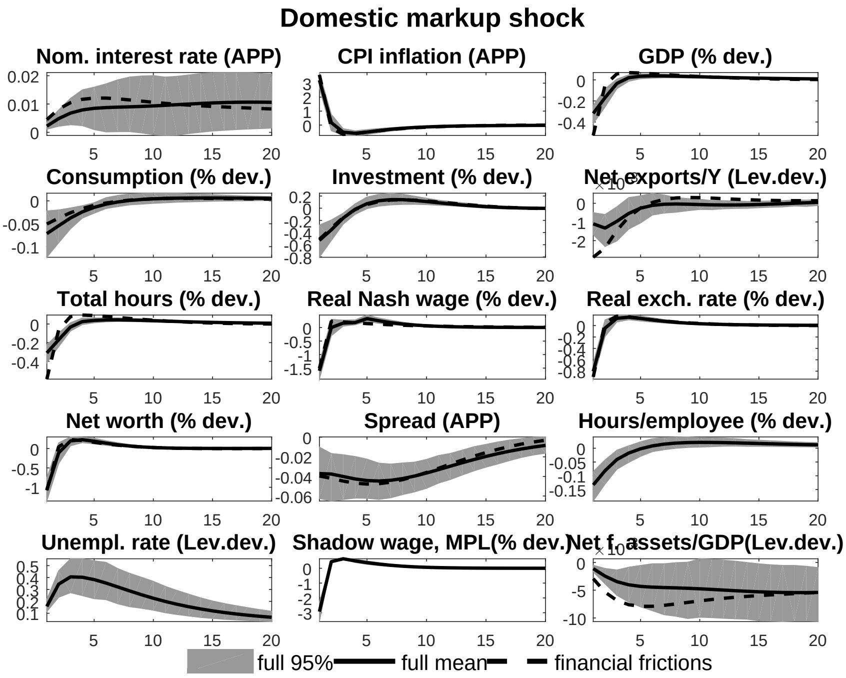

Figure 21: Impulse responses to the domestic markup shock, τ_t^d. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

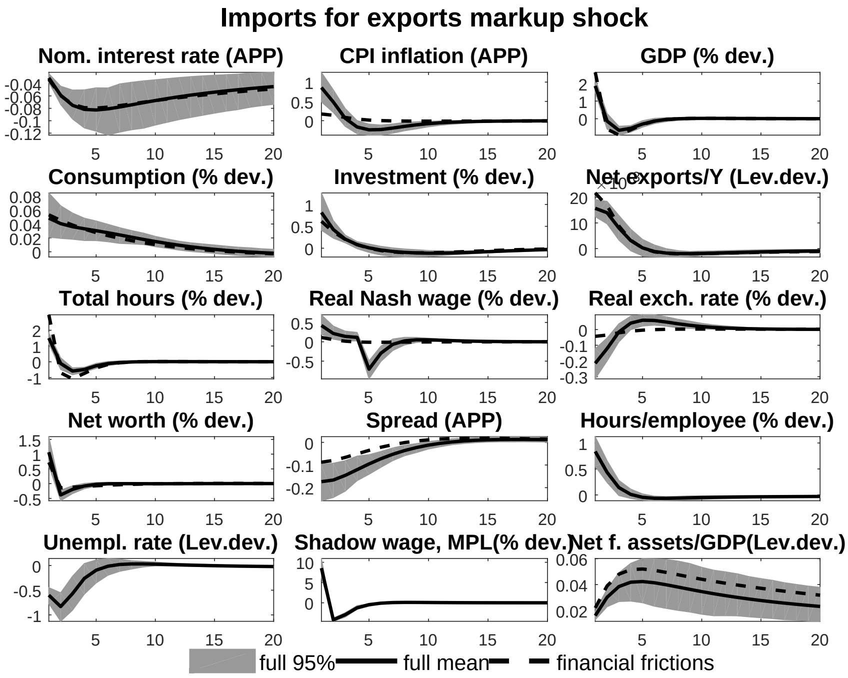

Figure 22: Impulse responses to the imports for exports markup shock, τ_t^{mx}. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

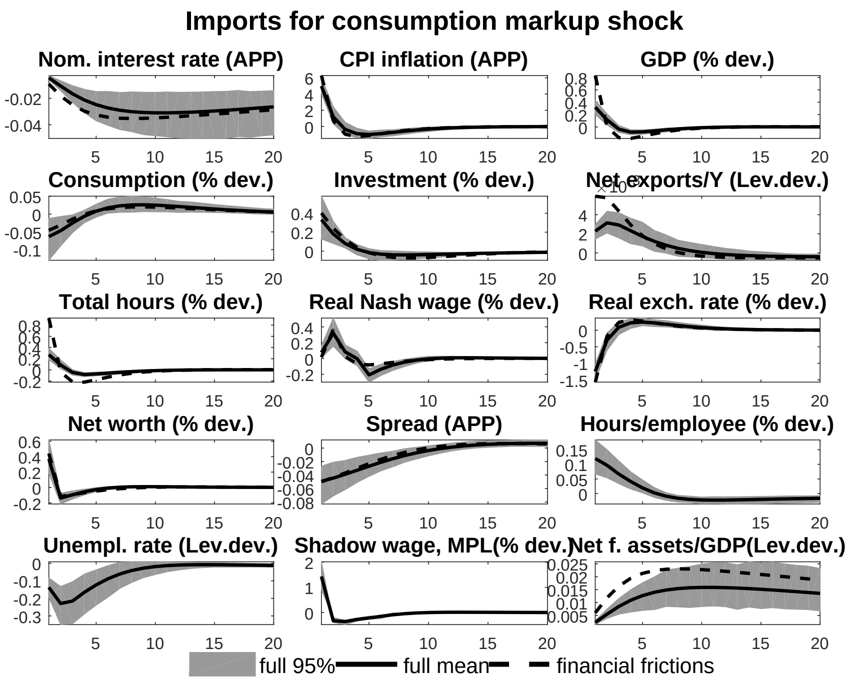

Figure 23: Impulse responses to the imports for consumption markup shock, τ_t^{mc}. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

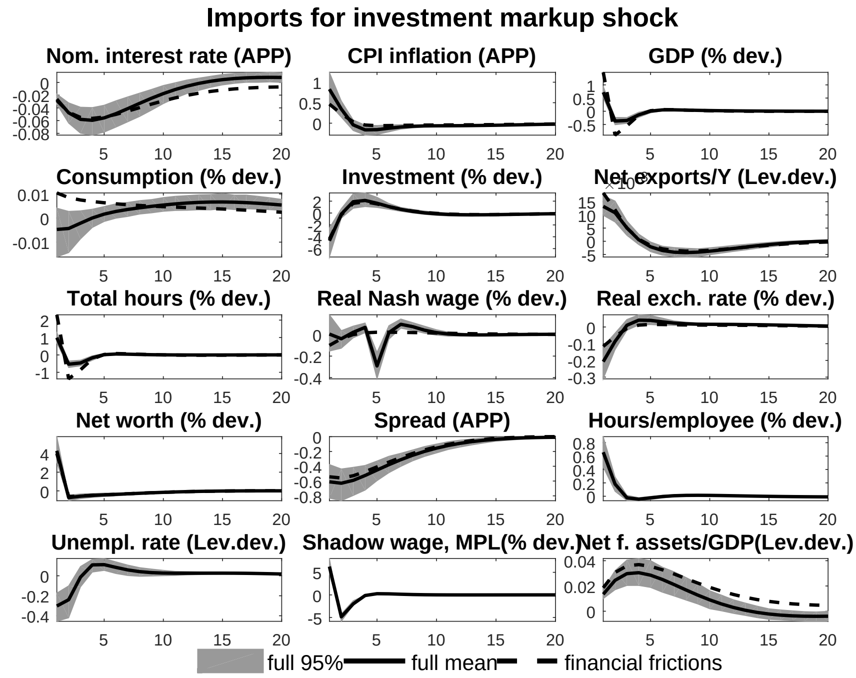

Figure 24: Impulse responses to the imports for investment markup shock, τ_t^{mi}. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

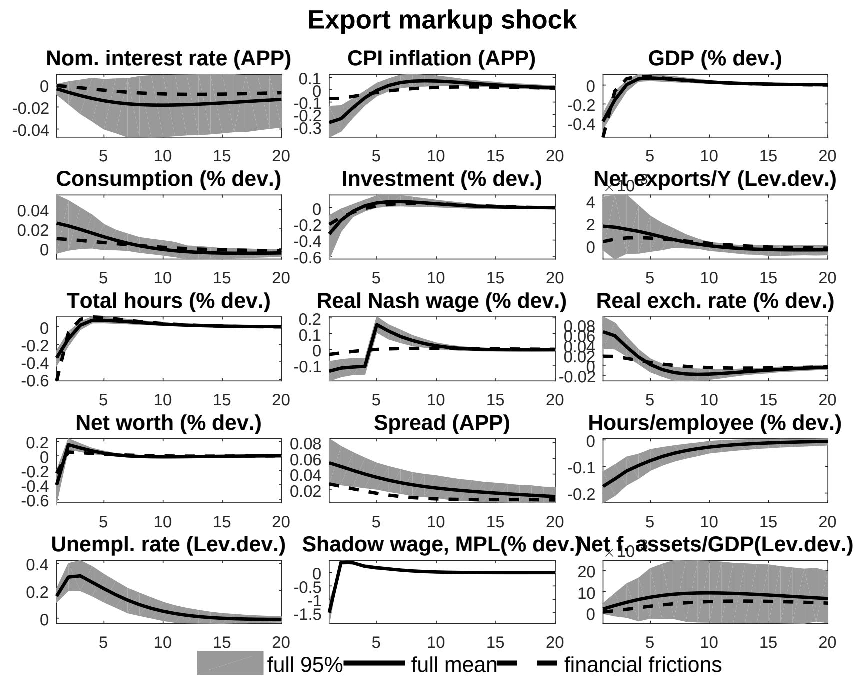

Figure 25: Impulse responses to the export markup shock, τ_t^{x}. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

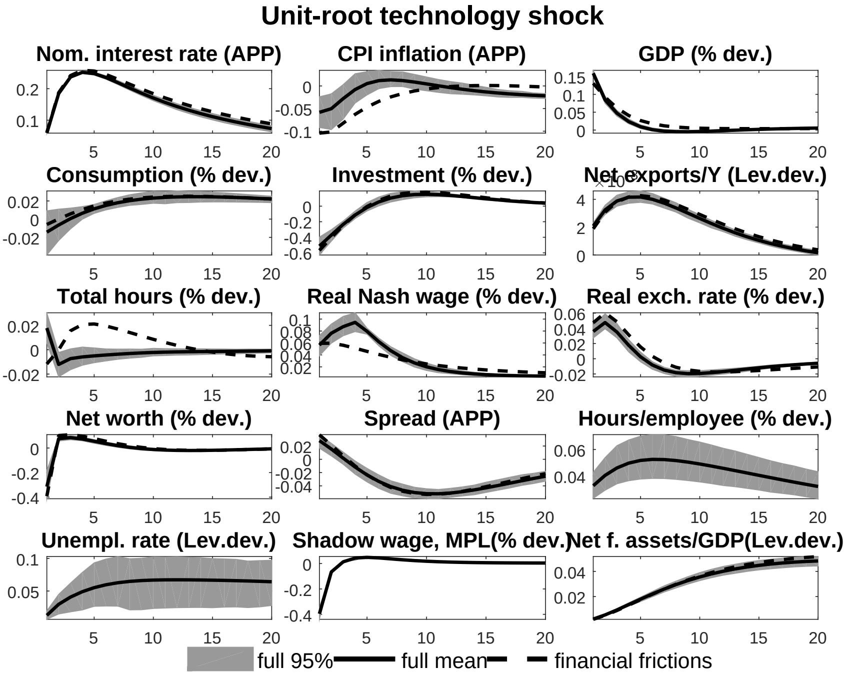

Figure 26: Impulse responses to the unit-root technology shock, $\mu_{z,t}$. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

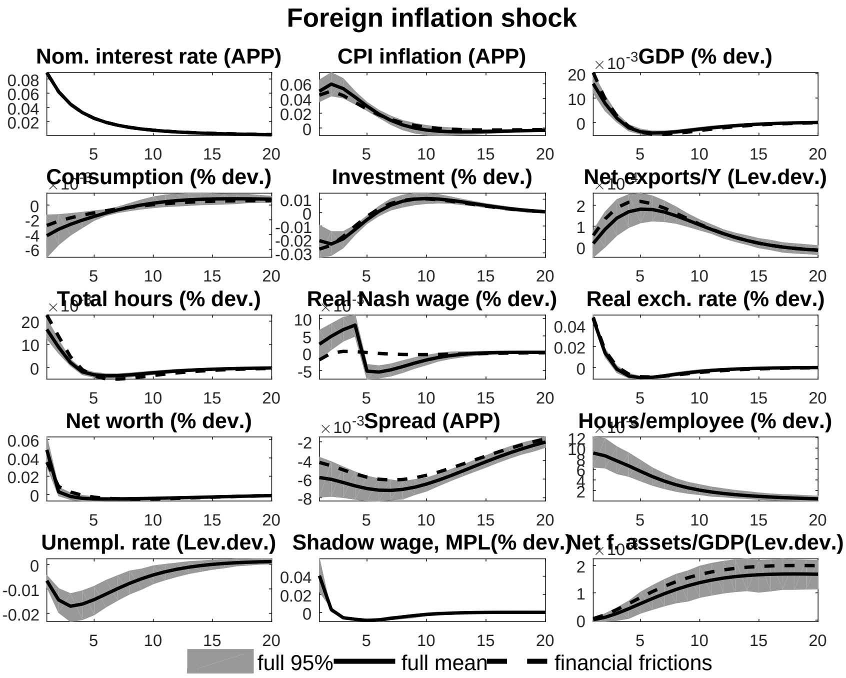

Figure 27: Impulse responses to the foreign inflation shock, $\epsilon_{\pi^*,t}$. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

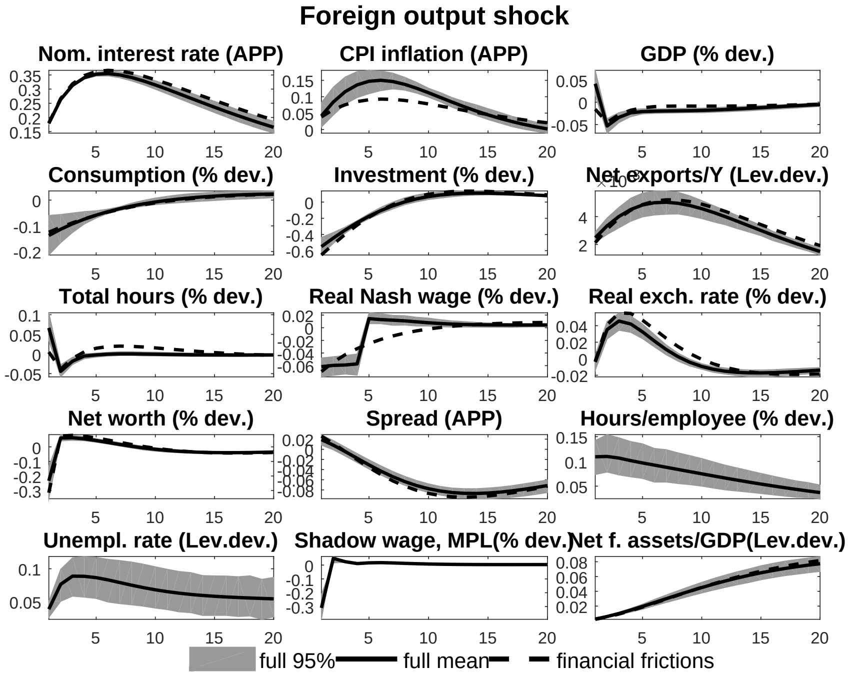

Figure 28: Impulse responses to the foreign output shock, $\epsilon_{y^*,t}$. The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev.dev.).

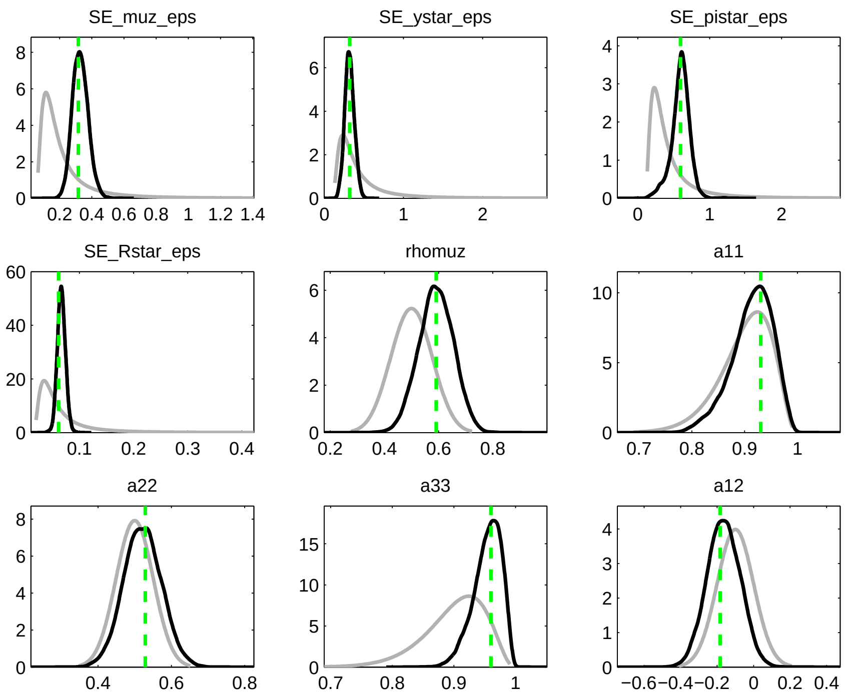

Figure 29: SVAR priors and posteriors.

Note: Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

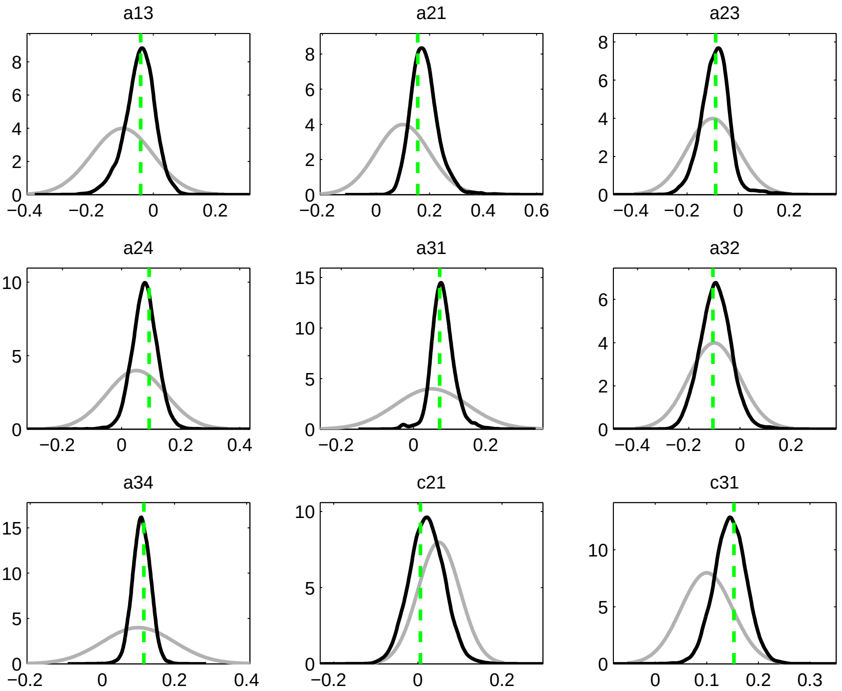

Figure 30: SVAR priors and posteriors (continued).

Note: Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

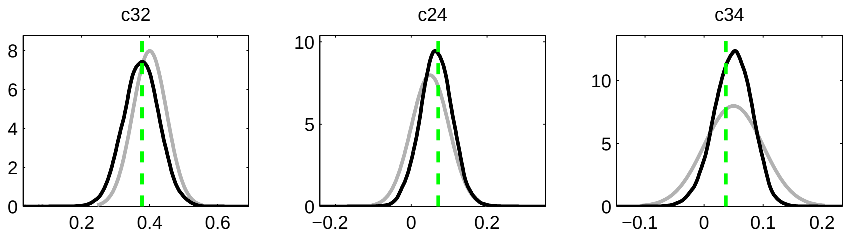

Figure 31: SVAR priors and posteriors (continued).

Note: Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

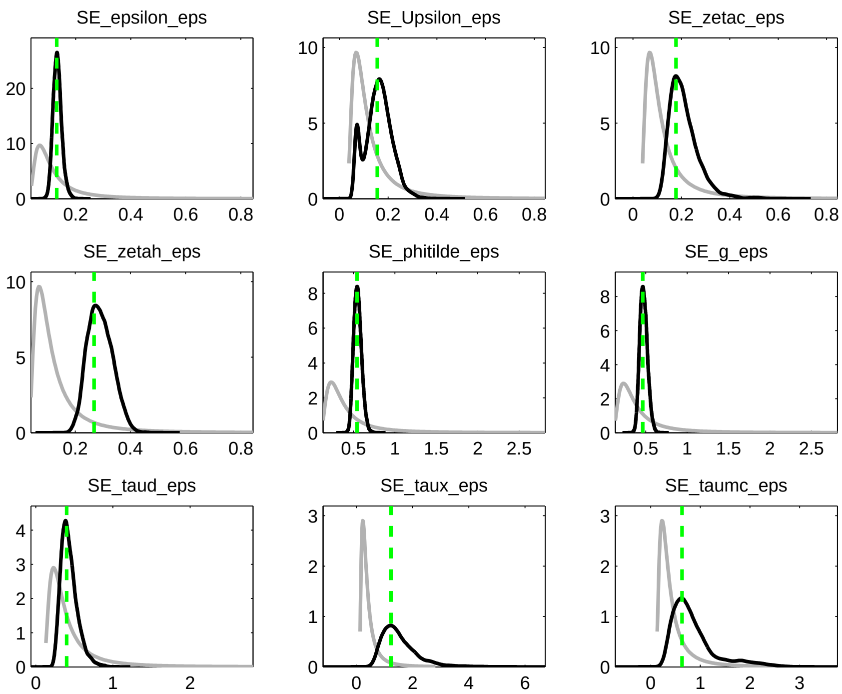

Figure 32: Priors and posteriors.

Note: Full model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

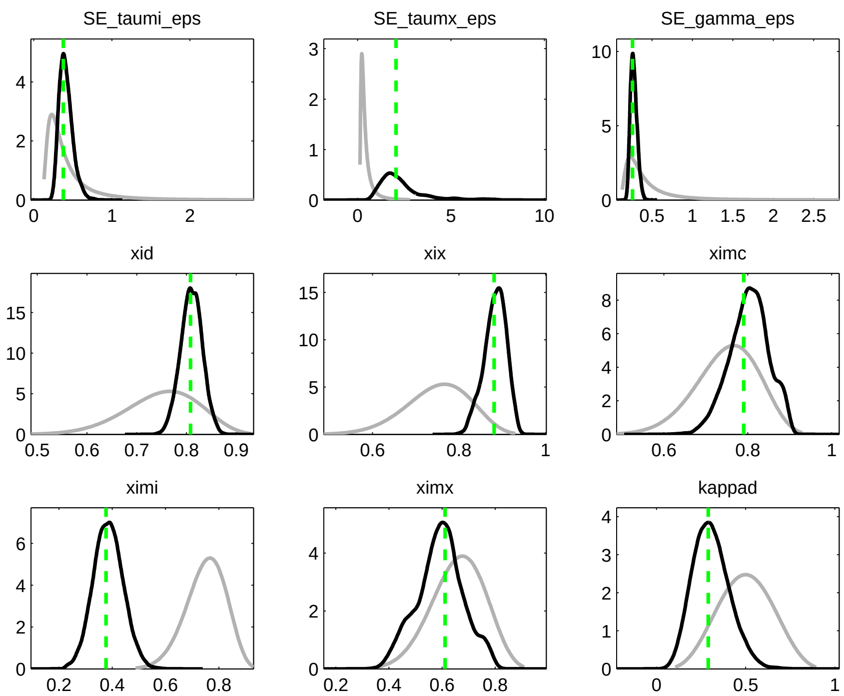

Figure 33: Priors and posteriors (continued).

Note: Full model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

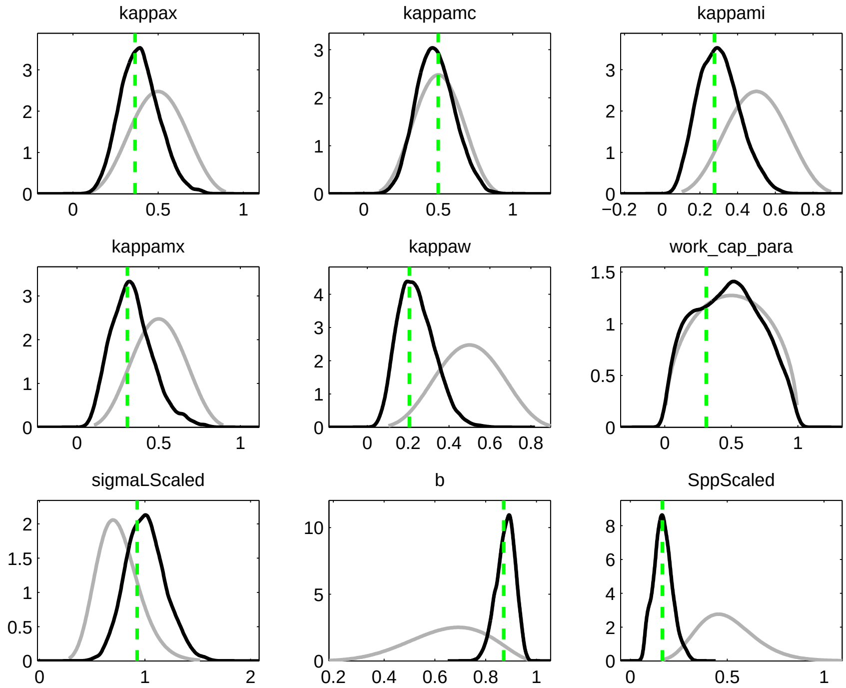

Figure 34: Priors and posteriors (continued).

Note: Full model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

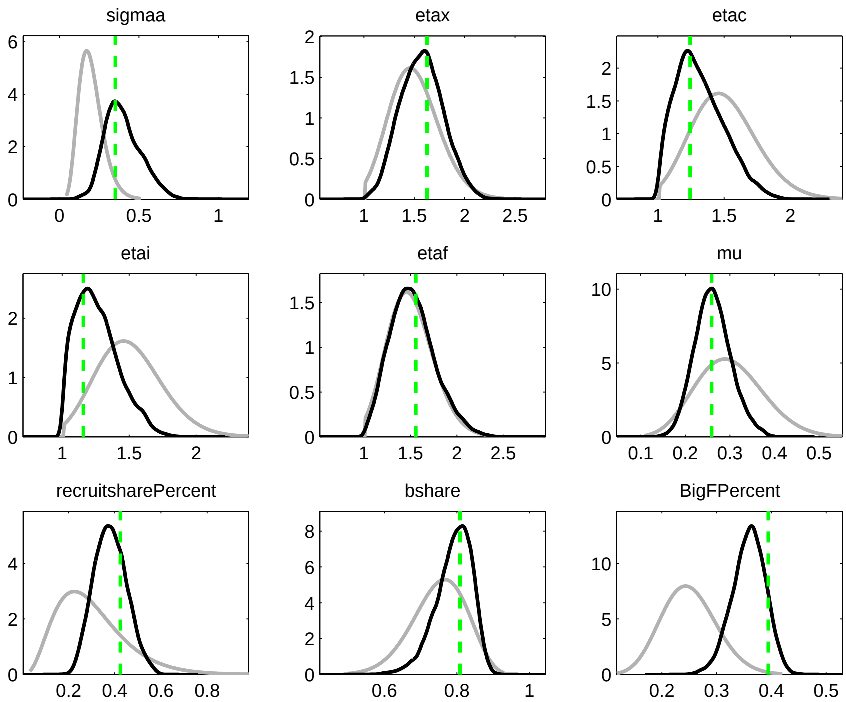

Figure 35: Priors and posteriors (continued).

Note: Full model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

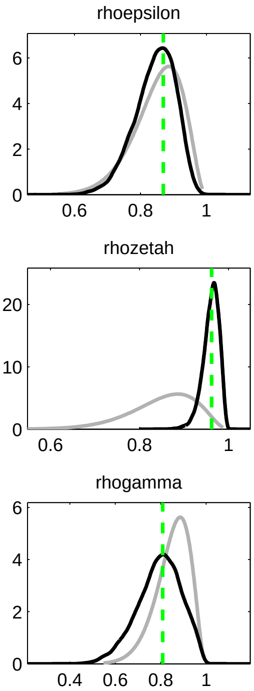

Figure 36: Priors and posteriors (continued).

Note: Full model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

Appendix C Online appendix on model details - not for publication

To save space, this section covers the details only about the labor block of the model. For the details on its core block and the financial frictions block, see the appendices in Buss (2014).

C.1 Employment frictions block

This section replaces the model of the labor market in the core block with the search and matching framework of Mortensen and Pissarides (1994), Hall (2005a,b), Shimer (2005,2012) as implemented in CTW. Endogenous separation of employees from their jobs is allowed, as in e.g. den Haan, Ramey and Watson (2000, henceforth dHRW). An implication of this modeling is increased volatility in unemployment. Also, Taylor type wage frictions are used instead of Calvo frictions due to the fact that empirically wage contracts normally have a fixed length and due to the ability to check that the wage always remain in the bargaining set in later periods of the wage contract.

C.1.1 Sketch of the model

The model adopts the Dixit-Stiglitz specification of homogeneous goods production. A representative competitive retail firm aggregates differentiated intermediate goods into a homogeneous good. Intermediate goods are supplied by monopolists who hire labor and capital services in competitive factor markets. The intermediate good firms are assumed to be subject to the same Calvo price setting friction as in the core block.

In the core block, the homogeneous labor services are supplied to the competitive labor market by labor contractors who combine the labor services supplied to them by households who monopolistically supply specialized labor services. In this model, the specialized labor services abstraction is not used. Instead, labor services are supplied by 'employment agencies' to the homogeneous labor market where they are bought by the intermediate goods producers. The change leaves the equilibrium conditions associated with the production of the homogeneous good unaffected. Key labor market activities - vacancy postings, layoffs, labor bargaining, setting the intensity of labor effort - are all carried out inside the employment agencies.

Each household is composed of many workers, each of which is in the labor force[27]. A worker begins the period either unemployed or employed with a particular employment agency. Unemployed workers do undirected search. They find a job with a particular agency with a probability that is proportional to the efforts made by the agency to attract workers. Workers are separated from employment agencies either exogenously, or because they are actively cut. Workers pass back and forth between unemployment and employment - there are no agency to agency transitions.

The events during the period in an employment agency take place in the following order. Each employment agency begins a period with a stock of workers. That stock is immediately reduced by exogenous separations and it is increased by new arrivals that reflect the agency's recruiting efforts in the previous period. Then, the economy's aggregate shocks are realized.

At this point, each agency's wage is set. The bargaining arrangement is atomistic, so that each worker bargains separately with a representative of the employment agency. The agencies are allocated permanently into N equal-sized cohorts and each period $1/N$ agencies establish a new wage by Nash bargaining. When a new wage is set, it evolves over the subsequent $N-1$ periods according to (C.1) and (C.2),

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \tag{C.1}$$

$$\tilde{\pi}_{w,t+1} = \left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_w-\varkappa_w} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}. \tag{C.2}$$

The wage negotiated in a given period covers all workers employed at an agency for each of the subsequent $N-1$ periods, even those that will not arrive until later.

Next, each worker draws an idiosyncratic productivity shock. A cutoff level of productivity is determined, and workers with lower productivity are laid off. From a technical point of view this modeling is symmetric to the modeling of entrepreneurial idiosyncratic risk and bankruptcy. Two mechanisms are considered by which the cutoff is determined. One is based on the total surplus of a given worker and the other is based purely on the employment agency's interest.

[27] In reality, the participation rate is also changing. To take that into account, I have tried to adjust the data on unemployment rate by the participation rate before fitting the model. The results show that the difference between the adjusted and the unadjusted unemployment rates are rather small compared to the total variance of the unemployment rate, so, for simplicity, I disregard the adjustment by the participation rate. Christiano, Trabandt and Walentin (2010) endogenizes labor force participation.

After the endogenous layoff decision, the employment agency posts vacancies and the intensive margin of labor supply is chosen efficiently by equating the marginal value of labor services to the employment agency with the marginal cost of providing it by the household. At this point employment agency supplies labor to the labor market.

We now describe the various labor market activities in greater detail. We begin with the decisions at the end of the period and work backwards to the bargaining problem because the bargaining problem internalizes everything that comes after.

C.1.2 Hours per worker

The intensive margin of labor supply is chosen to equate the value of labor services to the employment agency with the cost of providing it by the household. To explain the latter, consider the utility function of the household, which is a modified version of that of the benchmark model:

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \{\zeta_{t+l}^c \log(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l}^h A_L \left[\sum_{i=0}^{N-1} \frac{(\varsigma_{i,t+l})^{1+\sigma_L}}{1+\sigma_L} \left[1 - \mathcal{F}\left(\bar{a}_{t+l}^i; \sigma_{a,t+l}\right)\right] l_{t+l}^i \right] \}, \tag{C.3}$$

where $i \in \{0, \ldots, N-1\}$ indexes the cohort to which the employment agency belongs. The index $i = 0$ corresponds to the cohort whose employment agency renegotiates the wage in the current period, $i = 1$ corresponds to the cohort that renegotiated in the previous, and so on. The object l_t^i denotes the number of workers in cohort i, after exogenous separations and new arrivals from unemployment have occurred. Let a_t^i denote the idiosyncratic productivity shock drawn by a worker in cohort i. Then, $\bar{a}_t^i$ denotes the endogenously-determined cutoff such that all workers with $a_t^i < \bar{a}_t^i$ are laid off from the firm. Also, let

$$\mathcal{F}_t^i = \mathcal{F}(\bar{a}_t^i; \sigma_{a,t}) = \int_0^{\bar{a}_t^i} d\mathcal{F}(a; \sigma_{a,t}) \tag{C.4}$$

denote the cumulative distribution function of the idiosyncratic productivity. We assume that $\mathcal{F}$ is lognormal with $E(a) = 1$ and $V(\log(a)) = \sigma_a^2$. Accordingly,

$$\left[1 - \mathcal{F}_t^i\right] l_t^i \tag{C.5}$$

denotes the number of workers with an employment agency in the i^{th} cohort who survive the endogenous layoffs.

Let $\varsigma_{i,t}$ denote the number of hours supplied by a worker in the i^{th} cohort. The absence of the index a on $\varsigma_{i,t}$ reflects the assumption that each worker who survives endogenous layoffs in cohort i works the same number of hours, regardless of the realization of their idiosyncratic level of productivity. One justification for this is that any connection between hours and idiosyncratic productivity might induce workers to manipulated real or perceived productivity downwards. The disutility experienced by a worker that works $\varsigma_{i,t}$ hours is

$$\zeta_t^h A_L \frac{(\varsigma_{i,t})^{1+\sigma_L}}{1+\sigma_L}.$$

The household utility function (C.3) sums the disutility experienced by the workers in each cohort.

Although the individual worker's labor market experience - whether employed or unemployed - is determined by idiosyncratic shocks, each household has sufficiently many workers that the total fraction of workers employed,

$$L_t = \sum_{i=0}^{N-1} \left[1 - \mathcal{F}_t^i\right] l_t^i,$$

as well as the fractions allocated among the different cohorts, $[1 - \mathcal{F}_t^i] l_t^i$, $i = 0, \ldots, N-1$ are the same for each household. It is assumed that all the household's workers are supplied inelastically to the labor market, i.e. labor force participation is constant.

The household's current receipts arising from the labor market are

$$(1 - \tau^y)(1 - L_t)P_t b^u z_t^+ + \sum_{i=0}^{N-1} W_t^i \left[1 - \mathcal{F}_t^i\right] l_t^i \varsigma_{i,t} \frac{1 - \tau^y}{1 + \tau^w}, \tag{C.6}$$

where W_t^i is the nominal wage rate earned by workers in cohort $i = 0, \ldots, N-1$. The presence of the term involving b^u indicates the assumption that unemployed workers, $1 - L_t$, receive a pre-tax payment of $b^u z_t^+$ final consumption goods. These unemployment benefits are financed by lump sum taxes. As in the core model, there is a labor income tax τ_y and a payroll tax τ^w that affect the after-tax wage.

Let W_t denote the price, or 'shadow wage', received by employment agencies for supplying one unit of (effective) labor service to the intermediate goods producers. It represents the marginal gain to the employment agency that occurs when an individual worker increase time spent working by one (effective) unit. Because the employment agency is competitive in the supply of labor services, it takes W_t as given, and in equilibrium it coincides with the marginal product of labor and is connected to the marginal cost of the intermediate goods producers through (C.7) and (C.8),

$$mc_t = \tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k\right)^{\alpha} \left(\bar{w}_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t}, \tag{C.7}$$

$$mc_t = \tau_t^d \frac{\left(\mu_{\Psi,t}\right)^{\alpha} \bar{w}_t R_t^f}{\epsilon_t (1-\alpha) \left(\frac{k_{i,t}}{\mu_{z^+,t} H_{i,t}}\right)^{\alpha}}. \tag{C.8}$$

A real world interpretation is that it is the shadow value of an extra hour of work supplied by the human resources department to a firm.

It is assumed that hours per worker are chosen to equate the worker's marginal cost of working with the agency's marginal benefit:

$$W_t \mathcal{G}_t^i = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{\upsilon_t \frac{1-\tau^y}{1+\tau^w}} \tag{C.9}$$

for $i = 0, \ldots, N-1$, where $\mathcal{G}_t^i$ denotes expected productivity of workers who survive endogenous separation:

$$\mathcal{G}_t^i = \frac{\mathcal{E}_t^i}{1 - \mathcal{F}_t^i}, \tag{C.10}$$

where

$$\mathcal{E}_t^i := \mathcal{E}\left(\bar{a}_t^i; \sigma_{a,t}\right) := \int_{\bar{a}_t^i}^{\infty} a d\mathcal{F}(a; \sigma_{a,t}). \tag{C.11}$$

To understand the expression on the right of (C.9), note that the marginal cost, in utility terms, to an individual worker who increases hours worked by one unit is $\zeta_t^h A_L \varsigma_{i,t}^{\sigma_L}$. This is converted to currency units by dividing by the multiplier υ_t on the household's nominal budget constraint, and by the tax wedge $(1-\tau^y)/(1+\tau^w)$. The left side of (C.9) represents the increase in revenues to the employment agency from increasing hours worked by one unit. Division by $1-\mathcal{F}_t^i$ is required in (C.10) so that the expectation is relative to the distribution of a conditional on $a > \bar{a}_t^j$.

Labor intensity is potentially different across cohorts because $\mathcal{G}_t^i$ is indexed by cohort. When the wage rate is determined by Nash bargaining, it is taken into account that labor intensity is determined according to (C.9) and that workers will endogenously separate. Note that labor intensity as determined by (C.9) is efficient and unaffected by the negotiated wage and its rigidity.

C.1.3 Vacancies and the employment agency problem

The employment agency in the i^{th} cohort determines how many employees it will have in period $t+1$ by choosing vacancies v_t^i. The costs associated with v_t^i are

$$\frac{\kappa z_t^+}{\varphi}\left(\frac{Q_t^\iota v_t^i}{[1-\mathcal{F}_t^i]l_t^i}\right)^\varphi \left[1-\mathcal{F}_t^i\right] l_t^i$$

units of the domestic homogeneous good. The parameter $\varphi > 1$ determines the curvature of the cost function. Convex costs of adjusting the work force are assumed because linear costs would imply indeterminacy as dynamic wage dispersion imply that the costs of employees are heterogeneous across agencies, while the benefit of an additional employee is the same across agencies. $\kappa z_t^+/\varphi$ is a cost parameter which is assumed to grow at the same rate as the overall economic growth rate and, as noted above, $[1-\mathcal{F}_t^i]l_t^i$ denotes the number of employees in the i^{th} cohort after endogenous separations have occurred. Also, Q_t is the probability that a posted vacancy is filled, a quantity that is exogenous to an individual employment agency. If $\iota = 1$, costs are a function of the number of people hired, not the number of vacancy postings. Thus. $\iota = 1$ emphasize internal costs (e.g. training) of adjusting the work force, and not search costs. [Consider a shock that triggers an economic expansion and also produces a fall in the probability of filling a vacancy, Q_t. Then the expansion will be smaller in the version of the model that emphasizes search costs ($\iota = 0$) than in a version that emphasized internal costs ($\iota = 1$).]

To further describe the vacancy decisions of the employment agencies, their objective function is required. Begin by considering $F(l_t^0, \omega_t)$, the value function of the representative employment agency in the cohort $i = 0$ that negotiates its wage in the current period. The arguments of F are the agency's workforce after beginning-of-period exogenous separations and new arrivals, l_t^0, and an arbitrary value for the nominal wage rate, ω_t. That is, consider the value of the firm's problem after the wage rate has been set.

Suppose that the firm chooses a particular monotone transform of vacancy postings denoted by $\tilde{v}_t^i$:

$$\tilde{v}_t^i := \frac{Q_t^\iota v_t^i}{(1-\mathcal{F}_t^j)l_t^i}.$$

The agency's hiring rate, χ_t^i, is related to $\tilde{v}_t^i$ by

$$\chi_t^i = Q_t^{1-\iota}\tilde{v}_t^i. \tag{C.12}$$

To construct $F(l_t^0, \omega_t)$, one needs derive the law of motion of the firm's work force during the period of the wage contract. The time $t+1$ workforce of the representative agency in the i^{th} cohort at time t is denoted l_{t+1}^{i+1}. That workforce reflects the endogenous separations in period t as well as the exogenous separations and new arrivals at the start of period $t+1$. Let ρ denote the probability that an individual worker attached to an employment agency at the start of a period survives the exogenous separation. Then, given the hiring rate, χ_t^i,

$$l_{t+1}^{j+1} = (\chi_t^j + \rho)(1 - \mathcal{F}_t^j)l_t^j \tag{C.13}$$

for $j = 0, 1, \ldots, N-1$, with the understanding that $j = N$ is to be interpreted as $j = 0$.

The value function of the firm is

$$\begin{aligned} F(l_t^0, \omega_t) = &\sum_{j=0}^{N-1} \beta^j E_t \frac{\upsilon_{t+j}}{\upsilon_t} \max_{(\tilde{v}_{t+j}^i, \bar{a}_{t+j}^j)} \Big[\int_{\bar{a}_{t+j}^j}^{\infty} (W_{t+j}a - \Gamma_{t,j}\omega_t)\varsigma_{j,t+j} d\mathcal{F}(a) \\ &- P_{t+j}\frac{\kappa z_{t+j}^+}{\varphi}(\tilde{v}_{t+j}^j)^{\varphi}(1 - \mathcal{F}_{t+j}^j)\Big] l_{t+j}^j \\ &+ \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right), \end{aligned} \tag{C.14}$$

where l_t^j evolves according to (C.13), $\varsigma_{j,t}$ satisfies (C.9) and

$$\Gamma_{t,j} = \begin{cases} \tilde{\pi}_{w,t+j} \cdots \tilde{\pi}_{w,t+1}, & j > 0 \\ 1 & j = 0 \end{cases}, \tag{C.15}$$

where $\tilde{\pi}_{w,t}$ is defined in (C.2). Recall that W_{t+j} denotes the price paid to the employment agency for supplying one unit of labor to the intermediate goods producers in period $t+j$. The term $\Gamma_{t,j}\omega_t$ represents the wage rate in period $t+j$ given the wage rate was ω_t at time t and there have been no wage negotiations in periods $t+1$, $t+2$, $\cdots, t+j$. In (C.14), $\tilde{W}_{t+N}$ denotes the Nash bargaining wage that is negotiated in period $t+N$, which is when the next round of bargaining occurs. At time t the agency takes the state $t+N$-contingent function, $\tilde{W}_{t+N}$, as given. The vacancy decision of employment agencies solve the maximization problem in (C.14).

From (C.14), $F(l_t^0, \omega_t)$ is linear in l_t^0:

$$F(l_t^0, \omega_t) = J(\omega_t)l_t^0, \tag{C.16}$$

where $J(\omega_t)$ is not a function of l_t^0 and is the surplus that a firm bargaining in the current period enjoys from a match with an individual worker, when the current wage is ω_t. Although later in period workers become heterogeneous when they draw an idiosyncratic shock to productivity, the fact that the draw is i.i.d. over time means that workers are all identical at the time when (C.16) is evaluated.

C.1.4 Worker value functions

In order to discuss the endogenous separation decision as well as the bargaining problem, we must have the value function of the individual worker. For the bargaining problem, we require the worker's value function before he knows what his idiosyncratic productivity draw is. For the endogenous separation problem, we need to know the worker's value function after he knows that he has survived the endogenous separation. For both the bargaining and separation problem, we need to know the value of unemployment to the worker.

Let V_t^i denote the period t value of being a worker in an agency in cohort i after that worker has survived that period's endogenous separation:

$$\begin{aligned}V_t^i =&\Gamma_{t-i,i}\tilde{W}_{t-i}\varsigma_{i,t}\frac{1-\tau^y}{1+\tau^w} - A_L\frac{\zeta_t^h \varsigma_{i,t}^{1+\sigma_l}}{(1+\sigma_l)\upsilon_t} \\ &+ \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t}\left[\rho(1-\mathcal{F}_{t+1}^{i+1})V_{t+1}^{i+1} + (1-\rho+\rho\mathcal{F}_{t+1}^{i+1})U_{t+1}\right] \end{aligned} \tag{C.17}$$

for $i = 0, 1, \ldots, N-1$, where $\tilde{W}_{t-i}$ denotes the wage negotiated i periods in the past, and $\Gamma_{t-i,i}\tilde{W}_{t-i}$ represents the wage received in period t by workers in cohort i. The two terms after the equality in (C.17) represent a worker's period t flow utility, converted into units of currency. The term in square brackets in (C.17) correspond to utility in the possible period $t+1$ states in the world. With probability $\rho(1-\mathcal{F}_{t+1}^{i+1})$ the worker survives the exogenous and endogenous separations in period $t+1$, in which case its value function in $t+1$ is V_{t+1}^{i+1}. With the complementary probability, $1-\rho(1-\mathcal{F}_{t+1}^{i+1})$, the worker separates into unemployment in period $t+1$, and enjoys utility U_{t+1}.

The currency value of being unemployed in period t is

$$U_t = P_t z_t^+ b^u(1-\tau^y) + \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t}\left[f_t V_{t+1}^x + (1-f_t)U_{t+1}\right], \tag{C.18}$$

where f_t is the probability that an unemployment worker will land a job in period $t+1$, V_{t+1}^x is the period $t+1$ value function of a worker who knows that he has matched with an employment agency at the start of $t+1$ but does not know which one. In particular,

$$V_{t+1}^x = \sum_{i=0}^{N-1} \frac{\chi_t^i(1-\mathcal{F}_t^i)l_t^i}{m_t}\tilde{V}_{t+1}^{i+1}, \tag{C.19}$$

where total new matches at the start of period $t+1$, m_t, is given by

$$m_t = \sum_{j=0}^{N-1} \chi_t^j(1-\mathcal{F}_t^j)l_t^j. \tag{C.20}$$

In (C.19), $\chi_t^i(1-\mathcal{F}_t^i)l_t^i/m_t$ is the probability of finding a job in $t+1$ in an agency belonging to cohort i in period t. This is a proper probability distribution because it is positive for each i and it sums to unity by (C.20).

In (C.19), $\tilde{V}_{t+1}^{i+1}$ is the analog of V_{t+1}^{i+1}, except that the former is defined before the worker knows if he survives the endogenous productivity cut, while the latter is defined after survival. The superscript $i+1$ appears on $\tilde{V}_{t+1}^{i+1}$ because the probabilities in (C.19) refer to activities in a particular agency cohort in period t, while in period $t+1$ the index of that cohort is incremented by unity.

The definition of U_t in (C.18) is completed by giving the formal definition of $\tilde{V}_t^j$:

$$\tilde{V}_t^j = \mathcal{F}_t^j U_t + (1-\mathcal{F}_t^j) V_t^j, \tag{C.21}$$

that is, at the start of the period, the worker has probability $\mathcal{F}_t^j$ of returning to unemployment, and the complementary probability of surviving in the firm to work and receive a wage in period t.

C.1.5 Separation decision

Here we discuss the separation decision of a representative agency in the $j=0$ cohort which renegotiates the wage in the current period. The decisions of other cohorts are made in a similar way.

Just prior to the realization of idiosyncratic worker uncertainty, the number of workers attached to the representative agency in the $j=0$ cohort is l_t^0. Each of the workers in l_t^0 independently draws a productivity, a, from the cumulative distribution function, $\mathcal{F}$. The workers who draw a value of a below a productivity cutoff, $\bar{a}_t^0$, are separated from the agency and the rest remain. The productivity cutoff is selected by the representative agency taking as given all variables determined outside the agency. Alternative criteria for selecting $\bar{a}_t^0$ are considered. The different criteria correspond to different ways of weighting the surplus enjoyed by the agency and the surplus enjoyed by the workers, l_t^0, attached to the agency.

The aggregate surplus across all the l_t^0 workers in the representative agency is given by

$$(V_t^0 - U_t)(1-\mathcal{F}t^0) l_t^0. \tag{C.22}$$

To see this, note that each worker among the fraction $1-\mathcal{F}_t^0$ of workers with $a \geq \bar{a}_t^0$ who stay with the agency experiences the same surplus, $V_t^0 - U_t$. The fraction $\mathcal{F}_t^0$ of workers in l_t^0 who leave enjoys zero surplus. The object $\mathcal{F}_t^0$ is a function of $\bar{a}_t^0$ as indicated in (C.4).

The surplus enjoyed by the representative employment agency before idiosyncratic worker uncertainty is realized and when the workforce is l_t^0, is given by (C.14). According to (C.16), agency surplus per worker in l_t^0 is given by $J(\omega_t)$ having the following structure:

$$J(\omega_t) = \max_{\bar{a}_t^0} \tilde{J}(\omega_t; \bar{a}_t^0)(1-\mathcal{F}_t^0),$$

where

$$\tilde{J}(\omega_t; \bar{a}_t^0) = \max_{\tilde{v}_t^0} \left\{ \left(W_t \mathcal{G}_t^0 - \omega_t \right) \varsigma_{0,t} - P_t z_t^+ \frac{\kappa}{\varphi} \left(\tilde{v}_t^0 \right)^{\varphi} + \beta \frac{\upsilon_{t+1}}{\upsilon_t} \left(\chi_t^0 + \rho \right) J_{t+1}^1(\omega_t) \right\} \tag{C.23}$$

denotes the value to an agency in cohort 0 of an employee after endogenous separation has taken place. The terms χ_t^0 and $\tilde{v}_t^0$ are connected by (C.12). Thus, the surplus of the representative agency with workforce l_t^0 expressed as a function of an arbitrary value of $\bar{a}_t^0$ is

$$\tilde{J}(\omega_t; \bar{a}_t^0)(1-\mathcal{F}_t^0) l_t^0. \tag{C.24}$$

This expression displays the two ways how $\bar{a}_t^0$ impacts on firm profits: $\bar{a}_t^0$ affects the number of workers $1-\mathcal{F}_t^0$ employed in period t, as well as their average productivity and thereby the value to the employer of an employee, $\tilde{J}$. The impact of $\bar{a}_t^0$ on the number of workers can be deduced from (C.4). Although at first glance it may appear that the cutoff affects $\tilde{J}$ in several ways, in fact it only affects $\tilde{J}$ through the above two channels.

The surplus criterion governing the choice of $\bar{a}_t^0$ is specified to be a weighted sum of the worker surplus and employer surplus described above:

$$\left[s_w(V_t^0 - U_t) + s_e\tilde{J}(\omega_t;\bar{a}_t^0)\right](1-\mathcal{F}_t^0)l_t^0, \tag{C.25}$$

where parameters s_w, $s_e \in \{0,1\}$ allow for a variety of surplus measures. If $s_w = 0$ and $s_e = 1$ we have employer surplus. If $s_w = s_e = 1$ we have total surplus. Accordingly, the employer surplus model is the one in which $\bar{a}_t^0$ is chosen to optimize (C.25) with $s_w = 0$, $s_e = 1$, and the total surplus model is the one that optimizes (C.25) with $s_w = s_e = 1$. The first order condition for an interior optimum is

$$s_w V_t^{0\prime} + s_e\tilde{J}_{\bar{a}^0}(\omega_t;\bar{a}_t^0) = \left[s_w(V_t^0 - U_t) + s_e\tilde{J}(\omega_t;\bar{a}_t^0)\right]\frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0}, \tag{C.26}$$

according to which, $\bar{a}_t^0$ is selected to balance the impact on surplus along intensive and extensive margins. The expression on the left of the equality characterizes the impact on the intensive margin: the surplus per worker that survives the cut increases with $\bar{a}_t^0$, The expression on the right side of (C.26) captures the extensive margin, the loss of surplus associated with the $\mathcal{F}_t^{0\prime}/(1-\mathcal{F}_t^0)$ workers who do not survive the cut. The equations that characterize the choice of $\bar{a}_t^j$, $j = 1, \ldots, N-1$ are essentially the same as (C.26).

The expression (C.26) assumes an arbitrary wage outcome, ω_t. Next, we discuss the bargaining problem that determines a value for ω_t.

C.1.6 Bargaining problem

The bargaining occurs among a continuum of worker-agency representative pairs. Each bargaining session takes the outcomes of all other bargaining session as given. Because each bargaining session is atomistic, each session ignores its impact on the wage earned by workers arriving in the future during the contract. it is assumed that those future workers are simply paid the average of the outcome of all bargaining sessions. Since each bargaining problem is identical, the wage that solves each problem is the same and so the average wage coincides with the wage that solves the individual bargaining problem. Because each bargaining session is atomistic, it also ignores the impact of the wage bargaining on decisions line vacancies and separations, taken by the firm.

The Nash bargaining problem that determines the wage rate is a combination of the worker surplus and firm surplus:

$$\max_{\omega_t}\left(\tilde{V}_t^0 - U_t\right)^\eta J(\omega_t)^{1-\eta},$$

where the firm surplus, $J(\omega_t)$, reflects that the outside option of the firm in the bargaining problem is zero. Denote the wage that solves this problem by $\tilde{W}_t$.

Until now it was explicitly assumed that the negotiated wage paid by an employment agency which has renegotiated most recently i periods in the past is always inside the bargaining set, $[\underline{w}_t^i, \bar{w}_t^i]$, $i = 0, 1, \ldots, N-1$. In other words, the wage paid is not lower than the workers reservation wage and not higher than the wage an employment agency is willing to pay. The fact that we allow for endogenous separation when either total or employer surplus of a match is negative does not strictly guarantee that wages are in the bargaining set, i.e. that both employer and worker have a non-negative surplus of the match.

This completes the description of the employment friction representation of the labor market. This block also brings the three new shocks η_t, $\sigma_{m,t}$ and $\sigma_{a,t}$ into the model.

C.2 Scaling of variables and functional forms

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable Ψ_t is an investment-specific shock to technology and it is convenient to define the following combination of investment-specific and neutral technology:

$$\begin{aligned} z_t^+ &= \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &= \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{C.27}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+ \Psi_t$. Foreign and domestic inputs into the production of I_t (we denote these by I_t^d and I_t^d, respectively) are scaled by z_t^+. Consumption goods (C_t^m are imported intermediate consumption goods, C_t^d are domestically produced intermediate consumption goods, and C_t are final consumption goods) are scaled by z_t^+. Government spending, the real wage and real foreign assets are scaled by z_t^+. Exports (X_t^m are imported intermediate goods for use in producing exports and X_t are final export goods) are scaled by z_t^+. Also, υ_t is the shadow value in utility terms to the household of domestic currency and $\upsilon_t P_t$ is the shadow value of one unit of the homogeneous domestic good. The latter must be multiplied by z_t^+ to induce stationarity. $\tilde{P}_t$ is the within-sector relative price of a good. w_t denotes the ratio between the (Nash) wage paid to workers $\tilde{W}_t$ and the 'shadow wage' W_t paid by intermediate goods producers to the employment agencies in the employment friction block. Thus,

$$\begin{aligned}
k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t},\ i_t^d = \frac{I_t^d}{z_t^+},\ i_t = \frac{I_t}{z_t^+ \Psi_t},\ i_m^t = \frac{I_t^m}{z_t^+}, \\
c_t^m &= \frac{C_t^m}{z_t^+},\ c_t^d = \frac{C_t^d}{z_t^+},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ a_t := \frac{S_t A_{t+1}^*}{z_t^+ P_t}, \\
x_t^m &= \frac{X_t^m}{z_t^+},\ x_t = \frac{X_t}{z_t^+},\ \psi_{z^+,t} = \upsilon_t P_t z_t^+,\ (y_t =)\tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ w_t = \frac{\tilde{W}_t}{W_t}, \\
n_{t+1} &= \frac{\bar{N}_{t+1}}{z_t^+ P_t},\ w^e = \frac{W_t^e}{z_t^+ P_t}.
\end{aligned}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k,$$

where $P_{k',t}$ is in units of the domestic homogeneous good.

The nominal exchange rate is denoted by S_t and its growth rate is s_t:

$$s_t = \frac{S_t}{S_{t-1}}.$$

We define the following inflation rates:

$$\pi_t = \frac{P_t}{P_{t-1}},\ \pi_t^c = \frac{P_t^c}{P_{t-1}^c},\ \pi_t^* = \frac{P_t^*}{P_{t-1}^*},$$
$$\pi_t^i = \frac{P_t^i}{P_{t-1}^i},\ \pi_t^x = \frac{P_t^x}{P_{t-1}^x},\ \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}},$$

for $j = c, x, i$. Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of the domestic final consumption goods (i.e., the CPI), P_t^* is the price of a foreign homogeneous good, P_t^i is the price of the domestic final investment good and P_t^x is the price (in foreign currency units) of a final export good.

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good, P_t. When the price is denominated in foreign currency units, we divide by P_t^*, the price of the foreign homogeneous good. The exceptional case has to do with handling of the price of investment goods, P_t^i. This grows at a rate slower than P_t, and we therefore scale it by P_t/Ψ_t. Thus,

$$p_t^{m,x} = \frac{P_t^{m,x}}{P_t},\ p_t^{m,c} = \frac{P_t^{m,c}}{P_t},\ p_t^{m,i} = \frac{P_t^{m,i}}{P_t},$$
$$p_t^x = \frac{P_t^x}{P_t^*},\ p_t^c = \frac{P_t^c}{P_t},\ p_t^i = \frac{\Psi_t P_t^i}{P_t}. \tag{C.28}$$

Here, m, j means the price of an imported good which is subsequently used in the production of exports in the case $j = x$, in the production of the final consumption good in the case of $j = c$ and in the production of final investment good in the case of $j = i$. When there is just a single superscript the underlying good is a final good, with $j = x, c, i$ corresponding to exports, consumption and investment, respectively.

Functional forms

In the employment friction block we assume a log-normal distribution for idiosyncratic productivities of workers. This implies the following:

$$\mathcal{E}(\bar{a}_t^j;\sigma_{a,t}) = \int_{\bar{a}_t^j}^{\infty} ad\mathcal{F}(a;\sigma_{a,t}) = 1 - prob\left[v < \frac{\log(\bar{a}_t^j) + \frac{1}{2}\sigma_{a,t}^2}{\sigma_{a,t}} - \sigma_{a,t}\right], \qquad \text{(C.29)}$$

where $prob$ refers to the standard normal distribution, and (C.29) simply is (C.11) spelled out under this distributional assumption. Similarly eq. (C.4) becomes

$$\begin{aligned}\mathcal{F}(\bar{a}^j;\sigma_a) &= \int_0^{\bar{a}^j} d\mathcal{F}(a;\sigma_a) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\log(\bar{a}^j)+\frac{1}{2}\sigma_a^2}{\sigma}} \exp\left(\frac{-v^2}{2}\right)dv \\ &= prob\left[v < \frac{\log(\bar{a}^j) + \frac{1}{2}\sigma_a^2}{\sigma_a}\right]. \qquad \text{(C.30)}\end{aligned}$$

C.3 Equilibrium conditions for the employment frictions block

C.3.1 Labor hours

Scaling (C.9) by $P_t z_t^+$ yields

$$\bar{w}_t \mathcal{G}_t^i = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{\psi_{z^+,t}\frac{1-\tau^y}{1+\tau^w}} \qquad \text{(C.31)}$$

Note that the ratio

$$\frac{\mathcal{G}_t^i}{\varsigma_{i,t}^{\sigma_L}}$$

will be the same for all cohorts since no other variables in (C.31) are indexed by cohort.

C.3.2 Vacancies and the employment agency problem

An employment agency in the i^{th} cohort which does not renegotiate its wage in period t sets the period t wage, $W_{i,t}$, as in (C.1):

$$W_{i,t} = \tilde{\pi}_{w,t} W_{i-1,t-1},\ \tilde{\pi}_{w,t} := (\pi_{t-1})^{\kappa_w}(\bar{\pi}_t)^{1-\kappa_w-\varkappa_w}(\breve{\pi})^{\varkappa_w}(\mu_{z^+})^{\vartheta_w} \qquad \text{(C.32)}$$

for $i = 1, \ldots, N-1$ (note that an agency that was in the i^{th} cohort in period t was in cohort $i-1$ in period $t-1$) where κ_w, $\varkappa_w$, $\kappa_w + \varkappa_w \in (0,1)$.

After wages are set, employment agencies in cohort i decides endogenous separation, post vacancies to attract new workers in the next period and supply labor services, $l_t^i \varsigma_{i,t}$, into competitive labor markets. Simplifying,

$$
\begin{aligned}
F(l_t^0,\omega_t) = \sum_{j=0}^{N-1} \beta^j E_t \frac{\upsilon_{t+j}}{\upsilon_t} \max_{\tilde{v}_{t+j}^j} \Big[& \big(W_{t+j}\mathcal{E}_{t+j}^j - \Gamma_{t,j}\omega_t \left[1-\mathcal{F}_{t+j}^j\right]\big) \varsigma_{j,t+j} \\
& - P_{t+j}\frac{\kappa z_{t+j}^+}{\varphi}(\tilde{v}_t^j)^\varphi (1-\mathcal{F}_{t+j}^j)\Big] l_{t+j}^j \\
& + \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right).
\end{aligned} \tag{C.33}
$$

For convenience, we omit the expectation operator E_t below.

Writing out (C.33)

$$
\begin{aligned}
F(l_t^0,\omega_t) = \max_{\{v_{t+j}^j\}_{j=0}^{N-1}} \Big\{ & \Big[\big(W_t\mathcal{E}_t^0 - \omega_t(1-\mathcal{F}_t^0)\big)\varsigma_t - P_t\frac{\kappa z_t^+}{\varphi}(\tilde{v}_t^0)^\varphi(1-\mathcal{F}_t^0)\Big] l_t^0 \\
& + \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t}\left[(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t(1-\mathcal{F}_{t+1}^1))\varsigma_{t+1} - P_{t+1}\frac{\kappa z_{t+1}^+}{\varphi}(\tilde{v}_{t+1}^1)^\varphi(1-\mathcal{F}_{t+1}^1)\right] \\
& \times (\chi_t^0+\rho)[1-\mathcal{F}_t^0] l_t^0 \\
& + \beta^2 E_t \frac{\upsilon_{t+2}}{\upsilon_t}\left[(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t(1-\mathcal{F}_{t+2}^2))\varsigma_{t+2} - P_{t+2}\frac{\kappa z_{t+2}^+}{\varphi}(\tilde{v}_{t+2}^2)^\varphi(1-\mathcal{F}_{t+2}^2)\right] \\
& \times (\chi_{t+1}^1+\rho)(\chi_t^0+\rho)(1-\mathcal{F}_{t+1}^1)(1-\mathcal{F}_t^0) l_t^0 \\
& + \ldots + \\
& + \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right)\Big\}.
\end{aligned}
$$

$$
\begin{aligned}
J(\omega_t) = \max_{\{v_{t+j}^j\}_{j=0}^{N-1}} \Big\{ & (W_t\mathcal{E}_t^0 - \omega_t(1-\mathcal{F}_t^0))\varsigma_{0,t} - P_t z_t^+ \frac{\kappa}{\varphi}(\tilde{v}_t^0)^\varphi[1-\mathcal{F}_t^0] \\
& + \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t(1-\mathcal{F}_{t+1}^1))\varsigma_{1,t+1} - P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}(\tilde{v}_{t+1}^1)^\varphi(1-\mathcal{F}_{t+1}^1)\right]\times \\
& \times (\tilde{v}_t^0 Q_t^{1-\iota}+\rho)(1-\mathcal{F}_t^0) \\
& + \beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t(1-\mathcal{F}_{t+2}^2))\varsigma_{2,t+2} - P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}(\tilde{v}_{t+2}^2)^\varphi(1-\mathcal{F}_{t+2}^2)\right]\times \\
& \times (\tilde{v}_t^0 Q_t^{1-\iota}+\rho)(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho)(1-\mathcal{F}_{t+1}^1)[1-\mathcal{F}_t^0] \\
& + \ldots + \\
& + \beta^N\frac{\upsilon_{t+N}}{\upsilon_t} J(\tilde{W}_{t+N})(\tilde{v}_t^0 Q_t^{1-\iota}+\rho)(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho)\cdots(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho)\times \\
& \times (1-\mathcal{F}_{t+N-1}^{N-1})\cdots(1-\mathcal{F}_t^0)\Big\}.
\end{aligned} \tag{C.34}
$$

We derive optimal vacancy posting decisions of employment agencies by differentiating (C.34) with respect to $\tilde{v}_t^0$ and multiply the result by $(\tilde{v}_t^0 Q_t^{1-\iota}+\rho)/Q_t^{1-\iota}$ to obtain

$$
\begin{aligned}
0 = &- P_t z_t^+ \kappa (\tilde{v}_t^0)^{\varphi-1}[1-\mathcal{F}_t^0](\tilde{v}_t^0 Q_t^{1-\iota}+\rho)/Q_t^{1-\iota} \\
&+ \beta \frac{\upsilon_{t+1}}{\upsilon_t}\left[(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t[1-\mathcal{F}_{t+1}^1])\varsigma_{1,t+1} - P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}(\tilde{v}_{t+1}^1)^{\varphi}(1-\mathcal{F}_{t+1}^1)\right]\times \\
&\times (\tilde{v}_t^0 Q_t^{1-\iota}+\rho)[1-\mathcal{F}_t^0] \\
&+ \beta^2 \frac{\upsilon_{t+2}}{\upsilon_t}\left[(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t[1-\mathcal{F}_{t+2}^2])\varsigma_{2,t+2} - P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}(\tilde{v}_{t+2}^2)^{\varphi}(1-\mathcal{F}_{t+2}^2)\right]\times \\
&\times (\tilde{v}_t^0 Q_t^{1-\iota}+\rho)(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho)[1-\mathcal{F}_{t+1}^1][1-\mathcal{F}_t^0] \\
&+ \ldots + \\
&\beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J(\tilde{W}_{t+N})(\tilde{v}_t^0 Q_t^{1-\iota}+\rho)(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho)\cdots(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota}+\rho)\times \\
&\times [1-\mathcal{F}_{t+N-1}^{N-1}]\cdots[1-\mathcal{F}_t^0] \\
&= J(\omega_t) - (W_t\mathcal{E}_t^0 - \omega_t(1-\mathcal{F}_t^0))\varsigma_{0,t} + P_t z_t^+ \frac{\kappa}{\varphi}(\tilde{v}_t^0)^{\varphi}[1-\mathcal{F}_t^0] \\
&- P_t z_t^+ \kappa (\tilde{v}_t^0)^{\varphi-1}[1-\mathcal{F}_t^0](\tilde{v}_t^0 Q_t^{1-\iota}+\rho)/Q_t^{1-\iota}.
\end{aligned}
$$

Since the latter expression must be zero, we get [skip some math]

$$
J(\omega_t) = (W_t\mathcal{E}_t^0 - \omega_t(1-\mathcal{F}_t^0))\varsigma_{0,t} + P_t z_t^+ \kappa\left[\left(1-\frac{1}{\varphi}\right)(\tilde{v}_t^0)^{\varphi} + (\tilde{v}_t^0)^{\varphi-1}\frac{\rho}{Q_t^{1-\iota}}\right][1-\mathcal{F}_t^0].
$$

Next, we obtain simple expressions for the vacancy decisions from their FOCs for optimality. Multiplying the FOC for $\tilde{v}_{t+1}^1$ by

$$
(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho)\frac{1}{Q_{t+1}^{1-\iota}},
$$

substituting out the period $t+2$ and higher terms using the FOC for $\tilde{v}_t^0$ and rearranging [some math skipped]

$$
\frac{P_t z_t^+ \kappa (\tilde{v}_t^0)^{\varphi-1}}{Q_t^{1-\iota}} = \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\begin{array}{c} (W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t[1-\mathcal{F}_{t+1}^1])\varsigma_{1,t+1} \\ +P_{t+1}z_{t+1}^+\kappa(1-\mathcal{F}_{t+1}^1)\left[\left(1-\frac{1}{\varphi}\right)(\tilde{v}_{t+1}^1)^{\varphi} + (\tilde{v}_{t+1}^1)^{\varphi-1}\frac{\rho}{Q_{t+1}^{1-\iota}}\right]\end{array}\right].
$$

Continuing this way [some math skipped],

$$
\frac{P_{t+j} z_{t+j}^+ \kappa (\tilde{v}_{t+j}^j)^{\varphi-1}}{Q_{t+j}^{1-\iota}} = \beta\frac{\upsilon_{t+j+1}}{\upsilon_{t+j}}\left[\begin{array}{c} (W_{t+j+1}\mathcal{E}_{t+j+1}^{j+1} - \Gamma_{t,j+1}\omega_t[1-\mathcal{F}_{t+j+1}^{j+1}])\varsigma_{j+1,t+j+1} \\ +P_{t+j+1}z_{t+j+1}^+\kappa(1-\mathcal{F}_{t+j+1}^{j+1})\left[\begin{array}{c}\left(1-\frac{1}{\varphi}\right)(\tilde{v}_{t+j+1}^{j+1})^{\varphi} \\ +(\tilde{v}_{t+j+1}^{j+1})^{\varphi-1}\frac{\rho}{Q_{t+j+1}^{1-\iota}}\end{array}\right]\end{array}\right]
$$

for $j = 0, 1, \ldots, N-2$.

Now we consider the FOC for the optimality of $\tilde{v}_{t+N-1}^{N-1}$ [after some math]:

$$\frac{P_{t+N-1}z^+_{t+N-1}\kappa(\tilde{v}^{N-1}_{t+N-1})^{\varphi-1}}{Q^{1-\iota}_{t+N-1}} = \beta\frac{\upsilon_{t+N}}{\upsilon_{t+N-1}}\left[\begin{array}{c}(W_{t+N}\mathcal{E}^0_{t+N} - \tilde{W}_{t+N}[1-\mathcal{F}^0_{t+N}])\varsigma_{0,t+N} \\ +P_{t+N}z^+_{t+N}\kappa\left[\begin{array}{c}\left(1-\frac{1}{\varphi}\right)(\tilde{v}^0_{t+N})^{\varphi} \\ +(\tilde{v}^0_{t+N})^{\varphi-1}\frac{\rho}{Q^{1-\iota}_{t+N}}\end{array}\right](1-\mathcal{F}^0_{t+N})\end{array}\right]$$

The above FOCs apply over time to a group of agencies that bargain at date t. We now express the FOCs for a fixed date and different cohorts:

$$\begin{aligned}P_t z^+_t\kappa(\tilde{v}^j_t)^{\varphi-1}\frac{1}{Q^{1-\iota}_t} &= \beta\frac{\upsilon_{t+1}}{\upsilon_t}\Big[\Big(W_{t+1}\mathcal{E}^{j+1}_{t+1} - \Gamma_{t-j,j+1}\tilde{W}_{t-j}(1-\mathcal{F}^{j+1}_{t+1})\Big)\varsigma_{j+1,t+1} \\ &\quad + P_{t+1}z^+_{t+1}\kappa(1-\mathcal{F}^{j+1}_{t+1})\left(\left(1-\frac{1}{\varphi}\right)(\tilde{v}^{j+1}_{t+1})^{\varphi} + (\tilde{v}^{j+1}_{t+1})^{\varphi-1}\frac{\rho}{Q^{1-\iota}_{t+1}}\right)\Big]\end{aligned}$$

for $j = 0, \ldots, N-2$. Scaling by $P_t z^+_t$ yields

$$\begin{aligned}\kappa(\tilde{v}^j_t)^{\varphi-1}\frac{1}{Q^{1-\iota}_t} &= \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\Big[\big(\bar{w}_{t+1}\mathcal{E}^{j+1}_{t+1} - G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}(1-\mathcal{F}^{j+1}_{t+1})\big)\varsigma_{j+1,t+1} \\ &\quad + \kappa(1-\mathcal{F}^{j+1}_{t+1})\left(\left(1-\frac{1}{\varphi}\right)(\tilde{v}^{j+1}_{t+1})^{\varphi} + (\tilde{v}^{j+1}_{t+1})^{\varphi-1}\frac{\rho}{Q^{1-\iota}_{t+1}}\right)\Big] \qquad \text{(C.35)}\end{aligned}$$

for $j = 0, \ldots, N-2$, where

$$\begin{aligned}G_{t-i,i+1} &= \frac{\tilde{\pi}_{w,t+1}\cdots\tilde{\pi}_{w,t-i+1}}{\pi_{t+1}\cdots\pi_{t-i+1}}\left(\frac{1}{\mu_{z^+,t-i+1}}\right)\cdots\left(\frac{1}{\mu_{z^+,t+1}}\right),\ i\geq 0, \\ w_t &= \frac{\tilde{W}_t}{W_t},\ \bar{w}_t = \frac{W_t}{z^+_t P_t}, \qquad \text{(C.36)}\end{aligned}$$

and

$$G_{t,j} = \begin{cases}\frac{\tilde{\pi}_{w,t+j}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+j}\cdots\pi_{t+1}}\left(\frac{1}{\mu_{z^+,t+1}}\right)\cdots\left(\frac{1}{\mu_{z^+,t+j}}\right) & j>0 \\ 1 & j=0\end{cases} \qquad \text{(C.37)}$$

The scaled vacancy FOC of agencies that are in the last period of their contract is

$$\begin{aligned}\kappa(\tilde{v}^{N-1}_t)^{\varphi-1}\frac{1}{Q^{1-\iota}_t} &= \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\Big[\big(\bar{w}_{t+1}\mathcal{E}^0_{t+1} - w_{t+1}\bar{w}_{t+1}(1-\mathcal{F}^0_{t+1})\big)\varsigma_{0,t+1} \\ &\quad + \kappa(1-\mathcal{F}^0_{t+1})\left(\left(1-\frac{1}{\varphi}\right)(\tilde{v}^0_{t+1})^{\varphi} + (\tilde{v}^0_{t+1})^{\varphi-1}\frac{\rho}{Q^{1-\iota}_{t+1}}\right)\Big]. \qquad \text{(C.38)}\end{aligned}$$

C.3.3 Agency separation decisions

We start by considering the separation decision of a representative agency in the $j = 0$ cohort which renegotiates the wage in the current period. After that, we consider $j > 0$.

The separation decision of agencies that renegotiate the wage in the current period. We start by considering the impact of $\bar{a}_t^0$ on agency and worker surplus, respectively. The aggregate surplus across all the l_t^0 workers in the representative agency is given by (C.22). The object $\mathcal{F}_t^0$ is a function of $\bar{a}_t^0$ as indicated in (C.4). We denote its derivative by

$$\mathcal{F}_t^{j\prime} := \frac{d\mathcal{F}_t^j}{d\bar{a}_t^j} \tag{C.39}$$

for $j = 0, \ldots, N-1$. Where convenient, in this subsection we include expressions that apply to the representative agency in cohort $j > 0$ as well as to those in cohort $j = 0$. According to (C.9), $\bar{a}_t^0$ affects V_t^0 via its impact on hours worked, $\varsigma_{0,t}$. Hours worked is a function of $\bar{a}_t^0$ because $\mathcal{G}_t^0$ is (see (C.10), (C.9) and (C.17)). These observations about V_t^0 also apply to V_t^j for $j > 0$. Thus, differentiating (C.17), yields

$$V_t^{j\prime} := \frac{d}{d\bar{a}_t^j} V_t^j = \left[\Gamma_{t-j,j} \tilde{W}_{t-j} \frac{1-\tau^y}{1+\tau^w} - A_L \frac{\zeta_t \varsigma_{j,t}^{\sigma_L}}{\upsilon_t} \right] \varsigma_{j,t}' \tag{C.40}$$

where

$$\varsigma_{j,t}' := \frac{d\varsigma_{j,t}}{d\bar{a}_t^j} = \frac{1}{\sigma_L} (\varsigma_{j,t})^{1-\sigma_L} \frac{W_t \upsilon_t}{\zeta_t A_L} \frac{1-\tau^y}{1+\tau^w} \mathcal{G}_t^{j\prime} \tag{C.41}$$

and

$$\mathcal{G}_t^{j\prime} := \frac{d\mathcal{G}_t^j}{d\bar{a}_t^j}. \tag{C.42}$$

The counterpart to (C.41) in terms of scaled variables is

$$\varsigma_{j,t}' := \frac{1}{\sigma_L} (\varsigma_{j,t})^{1-\sigma_L} \frac{\bar{w}_t w_t \psi_{z^+,t}}{\zeta_t A_L} \frac{1-\tau^y}{1+\tau^w} \mathcal{G}_t^{j\prime} \tag{C.43}$$

The value of being unemployed, U_t, is not a function of the $\bar{a}_t^0$ chosen by the representative agency because U_t is determined by economy-wide aggregate variables such as the job finding rate (see (C.18)).

According to (C.16) agency surplus per worker in l_t^0 is given by $J(\omega_t)$ and this has the following representation:

$$J(\omega_t) = \max_{\bar{a}_t^0} \tilde{J}(\omega_t; \bar{a}_t^0)(1 - \mathcal{F}_t^0),$$

where $\tilde{J}(\omega_t; \bar{a}_t^0)$ is given by (C.23) and

$$
\begin{aligned}
J^{j+1}_{t+1}(\omega_t) = \max_{\{\bar{a}^i_{t+i},\tilde{v}^i_{t+i}\}_{i=j}^{N-1}} \Bigg\{ & \left[(W_{t+1}\mathcal{G}^{j+1}_{t+1} - \Gamma_{t-j,j+1}\omega_{t-j})\varsigma_{j+1,t+1} - P_{t+1}z^{+}_{t+1}\frac{\kappa}{\varphi}(\tilde{v}^{j+1}_{t+1})^{\varphi} \right] \\
& \times (1-\mathcal{F}^{j+1}_{t+1}) \\
& \beta\frac{\upsilon_{t+2}}{\upsilon_{t+1}} \left[(W_{t+2}\mathcal{G}^{j+2}_{t+2} - \Gamma_{t-j,j+2}\omega_{t-j})\varsigma_{j+2,t+2} - P_{t+2}z^{+}_{t+2}\frac{\kappa}{\varphi}(\tilde{v}^{j+2}_{t+2})^{\varphi} \right] \\
& \times (1-\mathcal{F}^{j+2}_{t+2})(\chi^{j+1}_{t+1}+\rho)(1-\mathcal{F}^{j+1}_{t+1}) \\
& + \ldots + \\
& + \beta^{N-j-1}\frac{\upsilon_{t+N-j}}{\upsilon_{t+1}} J\left(\tilde{W}_{t+N-j}\right)(\chi^{N-1}_{t+N-j-1}+\rho)(1-\mathcal{F}^{N-1}_{t+N-j-1})\cdots(\chi^{j+1}_{t+1}+\rho)(1-\mathcal{F}^{j+1}_{t+1}) \Bigg\}
\end{aligned}
\tag{C.44}
$$

for $j = 0$.

In (C.23) and (C.44), it is understood that χ^j_{t+j}, $\tilde{v}^j_{t+j}$ are connected by (C.12). Thus the surplus of the representative agency with workforce l^0_t expressed as a function of an arbitrary value of $\bar{a}^0_t$ is given by (C.24). Differentiation of $\tilde{J}$ with respect to $\bar{a}^j_t$ need only be concerned with the impact of $\bar{a}^j_t$ on $\mathcal{G}^j_t$ and $\varsigma_{j,t}$. Generalizing (C.23) to cohort j,

$$
\tilde{J}(\omega_{t-j};\bar{a}^j_t) = \max_{\tilde{v}^j_t} \left\{ (W_t\mathcal{G}^j_t - \Gamma_{t-j,j}\omega_{t-j})\varsigma_{j,t} - P_t z^{+}_t \frac{\kappa}{\varphi}(\tilde{v}^j_t)^{\varphi} + \beta\frac{\upsilon_{t+1}}{\upsilon_t}(\chi^j_t+\rho)J^{j+1}_{t+1}(\omega_{t-j}) \right\}
$$

Then,

$$
\tilde{J}_{\bar{a}^j}(\omega_{t-j};\bar{a}^j_t) := \frac{d\tilde{J}(\omega_{t-j};\bar{a}^j_t)}{d\bar{a}^j_t} = (W_t\mathcal{G}^j_t - \Gamma_{t-j,j}\omega_{t-j})\varsigma'_{j,t} + W_t\mathcal{G}^{j\prime}_t\varsigma_{j,t} \tag{C.45}
$$

where $\varsigma'_{j,t}$ and $\mathcal{G}^{j\prime}_t$ are defined in (C.41) and (C.42), respectively.

We now evaluate $\mathcal{F}^{j\prime}_t$ and $\mathcal{G}^{j\prime}_t$ for $j \geq 0$. It is assumed that productivity, a, is drawn from a log-normal distribution having the properties: $E(a) = 1$ and $V(\log(a)) = \sigma^2_a$. This assumption simplifies the analysis because analytic expressions are available for objects such as $\mathcal{F}^{j\prime}_t$, $\mathcal{G}^{j\prime}_t$. Although these expressions are readily available in the literature (e.g., in BGG), we derive them for completeness. it is easily verified that $\mathcal{F}$ has the following representation:[28]

$$
\mathcal{F}(\bar{a}^j;\sigma_a) = \frac{1}{\sigma_a\sqrt{2\pi}}\int_{-\infty}^{\log(\bar{a}^j)} e^x e^{\frac{-\left(x+\frac{1}{2}\sigma^2_a\right)^2}{2\sigma^2_a}} dx
$$

where $x = \log a$. Combining the exponential terms,

$$
\mathcal{F}(\bar{a}^j;\sigma_a) = \frac{1}{\sigma_a\sqrt{2\pi}}\int_{-\infty}^{\log(\bar{a}^j)} \exp\left[\frac{-\left(x-\frac{1}{2}\sigma^2_a\right)^2}{2\sigma^2_a}\right] dx
$$

[28] $E(a) = 1$ is when $E(\log(a)) = -\sigma^2_a/2$.

Making the change of variables,

$$v := \frac{x - \frac{1}{2}\sigma_a^2}{\sigma_a}$$

so that

$$dv = \frac{1}{\sigma_a} dx$$

and substituting into the expression for $\mathcal{F}$,

$$\mathcal{F}(\bar{a}^j; \sigma_a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\bar{a}^j) + \frac{1}{2}\sigma_a^2}{\sigma_a}} \exp(\frac{-v^2}{2}) dv.$$

This is just the standard normal cumulative distribution evaluated at $(\log(\bar{a}^j) + \frac{1}{2}\sigma_a^2)/\sigma_a$. Differentiating $\mathcal{F}$, we obtain expression for (C.39):

$$\mathcal{F}_t^{j\prime} = \frac{1}{\bar{a}^j \sigma_a \sqrt{2\pi}} \exp\left(- \frac{\left(\log(\bar{a}^j) + \frac{1}{2}\sigma_a^2\right)^2}{2\sigma_a^2} \right) \tag{C.46}$$

The object on the right of the equality is just the normal density with variance σ_a^2 and mean $-\sigma_a^2/2$ evaluated at $\log(\bar{a}^j)$ and divided by $\bar{a}^j$. From (C.11) yields

$$\mathcal{E}_t^{j\prime} = -\bar{a}_t^j \mathcal{F}_t^{j\prime}. \tag{C.47}$$

Differentiating (C.42),

$$\mathcal{G}_t^{j\prime} = \frac{\mathcal{E}_t^{j\prime}(1 - \mathcal{F}_t^j) + \mathcal{E}_t^j \mathcal{F}_t^{j\prime}}{[1 - \mathcal{F}_t^j]^2} \tag{C.48}$$

The surplus criterion governing the choice of $\bar{a}_t^0$ is (C.25). The FOC for an interior optimum is given by (C.26), which is reproduced here for convenience:

$$s_w V_t^{0\prime} + s_e \tilde{J}_{\bar{a}^0}\left(\tilde{W}_t; \bar{a}_t^0\right) = \left[s_w (V_t^0 - U_t) + s_e \tilde{J}\left(\tilde{W}_t; \bar{a}_t^0\right)\right] \frac{\mathcal{F}_t^{0\prime}}{1 - \mathcal{F}_t^0}$$

where the fact is used that the wage paid to workers in the bargaining period is denoted $\tilde{W}_t$. After substituting from (C.40) and (C.45),

$$\begin{aligned} s_w \left(\tilde{W}_t \frac{1 - \tau_y}{1 + \tau^w} - A_L \frac{\zeta_t \varsigma_{0,t}^{\sigma_L}}{\upsilon_t} \right) \varsigma_{0,t}' + s_e \left[\left(W_t \mathcal{G}_t^0 - \tilde{W}_t \right) \varsigma_{0,t}' + W_t \mathcal{G}_t^{0\prime} \varsigma_{0,t} \right] = \\ \left[s_w (V_t^0 - U_t) + s_e \tilde{J}\left(\tilde{W}_t; \bar{a}_t^0\right)\right] \frac{\mathcal{F}_t^{0\prime}}{1 - \mathcal{F}_t^0}. \end{aligned} \tag{C.49}$$

In scaled terms and dividing by $P_t z_t^+$ yields [some skipped math]:

$$s_w\left(w_t\bar{w}_t\frac{1-\tau^y}{1+\tau^w}-A_L\frac{\zeta_t\varsigma_{0,t}^{\sigma_L}}{\psi_{z^+,t}}\right)\varsigma_{0,t}'+s_e\bar{w}_t\left[\left(\mathcal{G}_t^0-w_t\right)\varsigma_{0,t}'+\mathcal{G}_t^{0\prime}\varsigma_{0,t}\right]=\left[\begin{array}{c}s_w\left(V_{z^+,t}^0-U_{z^+,t}\right)\\+s_e\tilde{J}_{z^+,t}^0\end{array}\right]\frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0} \tag{C.50}$$

The separation decision of agencies that renegotiated in previous periods. We now turn to the $\bar{a}_t^j$ decision, for $j=1,\ldots,N-1$. The representative agency that selects $\bar{a}_t^j$ is a member of the cohort of agencies that bargained j periods in the past. We denote the present discounted value of profits of the representative agency in cohort j by $F_t^j(\omega_{t-j})$:

$$\begin{aligned}\frac{F_t^j(l_t^j,\omega_{t-j})}{l_t^j} &:= J_t^j(\omega_{t-j})=\max_{\{\bar{a}_{t+i}^{j+i},\tilde{v}_{t+i}^{j+i}\}_{i=0}^{N-j-i}}\Bigg\{\left[(W_t\mathcal{G}_t^j-\Gamma_{t-j,j}\omega_{t-j})\varsigma_{j,t}-P_tz_t^+\frac{\kappa}{\varphi}(\tilde{v}_t^j)^{\varphi}\right]\\&\times(1-\mathcal{F}_t^j)\\&+\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[(W_{t+1}\mathcal{G}_{t+1}^{j+1}-\Gamma_{t-j,j+1}\omega_{t-j})\varsigma_{j+1,t+1}-P_{t+1}z_{t+1}^+\frac{\kappa}{\phi}(\tilde{v}_{t+1}^{j+1})^{\varphi}\right]\\&\times(1-\mathcal{F}_{t+1}^{j+1})(\chi_t^j+\rho)(1-\mathcal{F}_t^j)\\&+\ldots+\\&+\beta^{N-j}\frac{\upsilon_{t+N-j}}{\upsilon_t}J(\tilde{W}_{t+N-j})(\chi_{t+N-1-j}^{N-1}+\rho)(1-\mathcal{F}_{t+N-j-1}^{N-1})\cdots\\&\times(\chi_t^j+\rho)(1-\mathcal{F}_t^j)\Bigg\}\end{aligned}$$

Here, we use that $F_t^j(l_t^j,\omega_{t-j})$ is a proportional to l_t^j, as in the case $j=0$ considered in (C.16). In particular, $J_t^j(\omega_{t-j})$ is not a function of l_t^j and corresponds to the object in (C.44) with the time index t replaced by $t-j$. The term $J_t^j(\omega_{t-j})$ can be written as

$$J_t^j(\omega_{t-j})=\tilde{J}_t^j(\omega_{t-j};\bar{a}_t^j)(1-\mathcal{F}_t^j),$$

where

$$\tilde{J}(\omega_{t-j};\bar{a}_t^j)=(W_t\mathcal{G}_t^j-\Gamma_{t-j,j}\omega_{t-j})\varsigma_{j,t}-P_tz_t^+\frac{\kappa}{\varphi}(\tilde{v}_t^j)^{\varphi}+\beta\frac{\upsilon_{t+1}}{\upsilon_t}J_{t+1}^{j+1}(\omega_{t-j})(\chi_t^j+\rho)$$

from a generalization of (C.23) to $j=1,\ldots,N-1$.

in this way, we obtain an expression for agency surplus for agencies that have not negotiated for j periods which is symmetric to (C.24):

$$F_t^j(\omega_{t-j})=\tilde{J}_t^j(\omega_{t-j};\bar{a}_t^j)(1-\mathcal{F}_t^j)l_t^j. \tag{C.51}$$

The expression for total surplus is the analog of (C.25):

$$\left[s_w(V_t^j - U_t) + s_e\tilde{J}_t^j(\omega_{t-j};\bar{a}_t^j)\right](1-\mathcal{F}_t^j)l_t^j. \tag{C.52}$$

Differentiating,

$$s_w V_t^{j\prime} + s_e\tilde{J}_{\bar{a}^j}(\omega_{t-j};\bar{a}_t^j) = \left[s_w(V_t^j - U_t) + s_e\tilde{J}_t^j(\omega_{t-j};\bar{a}_t^j)\right]\frac{\mathcal{F}_t^{j\prime}}{1-\mathcal{F}_t^j} \tag{C.53}$$

which corresponds to (C.26). Here, $\tilde{J}_{\bar{a}^j}(\omega_{t-1};\bar{a}_t^j)$ is the analog of (C.45) with index 0 replaced by j. After substituting from the analogs for cohort j of (C.40), (C.45),

$$\begin{aligned}
&s_w\left(\Gamma_{t-j,j}\tilde{W}_{t-j}\frac{1-\tau^y}{1+\tau^w} - A_L\frac{\zeta_t\varsigma_{j,t}^{\sigma_L}}{\upsilon_t}\right)\varsigma_{j,t}' + s_e\left[\left(W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\tilde{W}_{t-j}\right)\varsigma_{j,t}' + W_t\mathcal{G}_t^{j\prime}\varsigma_{j,t}\right] \\
&\quad = \left[s_w(V_t^j - U_t) + s_e\tilde{J}\left(\tilde{W}_{t-j};\bar{a}_t^j\right)\right]\frac{\mathcal{F}_t^{j\prime}}{1-\mathcal{F}_t^j}.
\end{aligned}$$

Scaling analogously to (C.50) and plugging in $\tilde{W}_{t-j} = w_{t-j}\bar{w}_{t-j}P_{t-j}z_{t-j}^+$ and $\bar{w}_t z_t^+ P_t = W_t$, yields

$$\begin{aligned}
&s_w\left(G_{t-j,j}w_{t-j}\bar{w}_{t-j}\frac{1-\tau^y}{1+\tau^w} - A_L\frac{\zeta_t\varsigma_{j,t}^{\sigma_L}}{\psi_{z^+,t}}\right)\varsigma_{j,t}' + s_e\left[\left(\bar{w}_t\mathcal{G}_t^j - G_{t-j,j}\bar{w}_{t-j}w_{t-j}\right)\varsigma_{j,t}' + \bar{w}_t\mathcal{G}_t^{j\prime}\varsigma_{j,t}\right] \\
&\quad = \left[s_w(V_{z^+,t}^j - U_{z^+,t}) + s_e\tilde{J}_{z^+,t}^j\right]\frac{\mathcal{F}_t^{j\prime}}{1-\mathcal{F}_t^j}.
\end{aligned} \tag{C.54}$$

Finally, we need an explicit expression for $\tilde{J}(\tilde{W}_t;\bar{a}_t^j)$, or rather its scaled equivalent $\tilde{J}_{z^+,t}^j$. For this, use (C.44) to write out $J_{t+1}^{j+1}(\omega_{t-j})$ for $j = 1,\ldots,N$ and plug into (C.23):

$$\tilde{J}_t^j(\omega_{t-j};\bar{a}_t^j) = (W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\omega_{t-j})\varsigma_{j,t} - P_t z_t^+\frac{\kappa}{\varphi}(\tilde{v}_t^j)^\varphi + \beta\frac{\upsilon_{t+1}}{\upsilon_t}J_{t+1}^{j+1}(\omega_{t-j})(\chi_t^j + \rho)$$

Using (C.44),

$$\begin{aligned}
\tilde{J}_t^j(\omega_{t-j};\bar{a}_t^j) = {}& (W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\omega_{t-j})\varsigma_{j,t} - P_t z_t^+\frac{\kappa}{\varphi}(\tilde{v}_t^j)^\varphi + \beta\frac{\upsilon_{t+1}}{\upsilon_t}(\chi_t^j + \rho)\Big\{ \\
& \left[(W_{t+1}\mathcal{G}_{t+1}^{j+1} - \Gamma_{t-j,j+1}\omega_{t-j})\varsigma_{j+1,t+1} - P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}(\tilde{v}_{t+1}^{j+1})^\varphi\right](1-\mathcal{F}_{t+1}^{j+1}) \\
& + \beta\frac{\upsilon_{t+2}}{\upsilon_{t+1}}\left[(W_{t+2}\mathcal{G}_{t+2}^{j+2} - \Gamma_{t-j,j+2}\omega_{t-j})\varsigma_{j+2,t+2} - P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}(\tilde{v}_{t+2}^{j+2})^\varphi\right] \\
& \times(1-\mathcal{F}_{t+2}^{j+2})(\chi_{t+1}^{j+1} + \rho)(1-\mathcal{F}_{t+1}^{j+1}) \\
& + \ldots + \\
& + \beta^{N-j-1}\frac{\upsilon_{t+N-j}}{\upsilon_{t+1}}J\left(\tilde{W}_{t+N-j}\right)(\chi_{t+N-j-1}^{N-1} + \rho)(1-\mathcal{F}_{t+N-j-1}^{N-1})\cdots \\
& \times(\chi_{t+1}^{j+1} + \rho)(1-\mathcal{F}_{t+1}^{j+1})\Big\}
\end{aligned}$$

for $j = 0, \ldots, N-1$. Plugging in for $\omega_{t-j} = \tilde{W}_{t-j} = w_{t-j}\bar{w}_{t-j}P_{t-j}z^+_{t-j}$, scaling and rearranging [some math skipped],

$$
\begin{aligned}
\tilde{J}^j_{z^+,t}(\tilde{W}_{t-j};\bar{a}^j_t) := \frac{\tilde{J}^j(\tilde{W}_t;\bar{a}^j_t)}{P_t z^+_t} &= (\bar{w}_t\mathcal{G}^j_t - G_{t-j,j}w_{t-j}\bar{w}_{t-j})\varsigma_{j,t} - \frac{\kappa}{\varphi}(\tilde{v}^j_t)^{\varphi} \\
&+ \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}(\chi^j_t+\rho)\Big\{ \\
&\left[(\bar{w}_{t+1}\mathcal{G}^{j+1}_{t+1} - G_{t-j,j+1}w_{t-j}\bar{w}_{t-j})\varsigma_{j+1,t+1} - \frac{\kappa}{\varphi}(\tilde{v}^{j+1}_{t+1})^{\varphi}\right](1-\mathcal{F}^{j+1}_{t+1}) \\
&+ \beta\frac{\psi_{z^+,t+2}}{\psi_{z^+,t+1}}\left[(\bar{w}_{t+2}\mathcal{G}^{j+2}_{t+2} - G_{t-j,j+2}w_{t-j}\bar{w}_{t-j})\varsigma_{j+2,t+2} - \frac{\kappa}{\varphi}(\tilde{v}^{j+2}_{t+2})^{\varphi}\right] \\
&\times (1-\mathcal{F}^{j+2}_{t+2})(\chi^{j+1}_{t+1}+\rho)(1-\mathcal{F}^{j+1}_{t+1}) \\
&+ \ldots + \\
&+ \beta^{N-j-1}\frac{\psi_{z^+,t+N-j}}{\psi_{z^+,t+1}}J_{z^+,t+N-j}(\chi^{N-1}_{t+N-j-1}+\rho)(1-\mathcal{F}^{N-1}_{t+N-j-1})\cdots \\
&\times (\chi^{j+1}_{t+1}+\rho)(1-\mathcal{F}^{j+1}_{t+1})\Big\}
\end{aligned}
\tag{C.55}
$$

C.3.4 Bargaining problem

The FOC associated with the Nash bargaining problem, after division by $z^+_t P_t$, is

$$\eta_t V_{w,t} J_{z^+,t} + (1-\eta_t)[V^0_{z^+,t} - U_{z^+,t}]J_{w,t} = 0 \tag{C.56}$$

The following is an expression for J_t evaluated at $\omega_t = \tilde{W}_t$, in terms of scaled variables

$$
\begin{aligned}
J_{z^+,t} &= \sum_{j=0}^{N-1}\beta^j\frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}\left[(\bar{w}_{t+j}\mathcal{G}^j_t - G_{t,j}w_t\bar{w}_t)\varsigma_{j,t+j} - \frac{\kappa}{\varphi}(\tilde{v}^j_{t+j})^{\varphi}\right]\Omega^j_{t+j} \\
&+ \beta^N\frac{\psi_{z^+,t+N}}{\psi_{z^+,t}}J_{z^+,t+N}\frac{\Omega^N_{t+N}}{1-\mathcal{F}^0_{t+N}}.
\end{aligned}
\tag{C.57}
$$

We also require the derivative of J with respect to ω_t, i.e. the marginal surplus of the employment agency with respect to the negotiated wage. By the envelope condition, we can ignore the impact of a change in ω_t on endogenous separations and vacancy decisions, and only be concerned with the direct impact of ω_t on J. Taking the derivative of (C.34),

$$
\begin{aligned}
J_{w,t} &= -(1-\mathcal{F}^0_t)\varsigma_{0,t} \\
&- \beta\frac{\upsilon_{t+1}}{\upsilon_t}\Gamma_{t,1}\varsigma_{1,t+1}(\chi^0_t+\rho)(1-\mathcal{F}^1_{t+1})(1-\mathcal{F}^0_t) \\
&- \beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\Gamma_{t,2}\varsigma_{2,t+2}(\chi^0_t+\rho)(\chi^1_{t+1}+\rho)(1-\mathcal{F}^2_{t+2})(1-\mathcal{F}^1_{t+1})(1-\mathcal{F}^0_t) \\
&- \ldots - \beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\Gamma_{t,N-1}\varsigma_{N-1,t+N-1}(\chi^0_t+\rho)(\chi^1_{t+1}+\rho)\cdots(\chi^{N-2}_{t+1}+\rho)\times \\
&\times (1-\mathcal{F}^{N-1}_{t+N-1})\cdots(1-\mathcal{F}^0_t).
\end{aligned}
$$

Let

$$\Omega_{t+j}^{j} = \begin{cases} (1-\mathcal{F}_{t+j}^{j}) \prod_{l=0}^{j-1} (\chi_{t+l}^{l}+\rho)(1-\mathcal{F}_{t+l}^{l}) & j>0 \\ 1-\mathcal{F}_{t}^{0} & j=0 \end{cases} \tag{C.58}$$

It is convenient to express this in recursive form:

$$\Omega_t^0 = 1-\mathcal{F}_t^0,\ \Omega_{t+1}^1 = (1-\mathcal{F}_{t+1}^1)(\chi_t^0+\rho)\overbrace{(1-\mathcal{F}_t^0)}^{\Omega_t^0},$$

$$\Omega_{t+2}^2 = (1-\mathcal{F}_{t+2}^2)(\chi_{t+1}^1+\rho)\overbrace{(\chi_t^0+\rho)(1-\mathcal{F}_t^0)(1-\mathcal{F}_{t+1}^1)}^{\Omega_{t+1}^1},$$

so that

$$\Omega_{t+j}^{j} = (1-\mathcal{F}_{t+j}^{j})(\chi_{t+j-1}^{j-1}+\rho)\Omega_{t+j-1}^{j-1}$$

for $j = 1, 2, \ldots$. It is convenient to define these objects at date t as a function of variables dated t and earlier for the purposes of implementing these equations in Dynare:

$$\Omega_t^0 = 1-\mathcal{F}_t^0,\ \Omega_t^1 = (1-\mathcal{F}_t^1)(\chi_{t-1}^0+\rho)\overbrace{(1-\mathcal{F}_{t-1}^0)}^{\Omega_{t-1}^0},$$

$$\Omega_t^2 = (1-\mathcal{F}_t^2)(\chi_{t-1}^1+\rho)\overbrace{(\chi_{t-2}^0+\rho)(1-\mathcal{F}_{t-2}^0)(1-\mathcal{F}_{t-1}^1)}^{\Omega_{t-1}^1},$$

so that

$$\Omega_t^j = (1-\mathcal{F}_t^j)(\chi_{t-1}^{j-1}+\rho)\Omega_{t-1}^{j-1}.$$

Then, in terms of scaled variables,

$$J_{w,t} = -\sum_{j=0}^{N-1} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} G_{t,j}\Omega_{t+j}^{j}\varsigma_{j,t+j}. \tag{C.59}$$

Scaling V_t^i by $P_t z_t^+$,

$$\begin{aligned} V_{z^+,t}^{i} &= G_{t-i,i} w_{t-i} \bar{w}_{t-i} \varsigma_{i,t} \frac{1-\tau^y}{1+\tau^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L)\psi_{z^+,t}} \\ &+ \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[\rho(1-\mathcal{F}_{t+1}^{i+1})V_{z^+,t+1}^{i+1} + (1-\rho+\rho\mathcal{F}_{t+1}^{i+1})U_{z^+,t+1}\right] \end{aligned} \tag{C.60}$$

for $i = 0, 1, \ldots, N-1$, where

$$\frac{V_t^i}{P_t z_t^+} = V_{z^+,t}^i,\; U_{z^+,t+1} = \frac{U_{t+1}}{P_{t+1} z_{t+1}^+}.$$

in our analysis of the Nash bargaining problem, we must have the derivative of V_t^0 with respect to the wage rate. To define this derivative, it is useful to have

$$\mathcal{M}_{t+j} = (1 - \mathcal{F}_t^0) \cdots (1 - \mathcal{F}_{t+j}^j) \tag{C.61}$$

for $j = 0, \ldots, N-1$. Then the derivative of V^0, denoted as $V_w^0(\omega_t)$, is

$$\begin{aligned} V_w^0(\omega_t) &= E_t \sum_{j=0}^{N-1} (\beta\rho)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau^y}{1+\tau^w} \Gamma_{t,j} \frac{\upsilon_{t+j}}{\upsilon_t} \\ &= E_t \sum_{j=0}^{N-1} (\beta\rho)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau^y}{1+\tau^w} G_{t,j} \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} \end{aligned} \tag{C.62}$$

Note that ω_t has no impact on the intensity of labor effort. This is determined by (C.31), independent of the wage rate paid to workers.

Scaling (C.18),

$$U_{z^+,t} = b^u(1-\tau^y) + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} [f_t V_{z^+,t+1}^x + (1-f_t) U_{z^+,t+1}]. \tag{C.63}$$

This value function applies to any unemployed worker, whether they got that way because they were unemployed in the previous period and did not find a job, or they arrived into unemployment because of an exogenous separation, or because they arrived because of an endogenous separation.

C.3.5 Resource constraint in the full model

It is assumed that the posting of vacancies uses the homogeneous domestic good. We leave the production technology equation, (C.64),

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^*,t}} k_t \right)^{\alpha} \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1-\alpha} - \phi \right], \tag{C.64}$$

unchanged, and we alter the resource constraint:

$$\begin{aligned} y_t - \frac{\kappa}{2} \sum_{j=0}^{N-1} (\tilde{v}_t^j)^2 (1 - \mathcal{F}_t^j) l_t^j &= g_t + c_t^d + i_t^d \\ &+ (R_t^x)^{\eta_x} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x)(p_t^x)^{-\eta_f} y_t^*. \end{aligned} \tag{C.65}$$

Measured GDP is y_t adjusted for both recruitment (hiring) costs and capital utilization costs:

$$gdp_t = y_t - \frac{\kappa}{2}\sum_{j=0}^{N-1}(\tilde{v}_t^j)^2(1-\mathcal{F}_t^j)l_t^j - (p_t^i)^{\eta_i}\left(a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i)$$

C.3.6 Final equilibrium conditions

Total job matches must also satisfy the following matching function:[29]

$$m_t = \sigma_m(1-L_t)^{\sigma}v_t^{1-\sigma} \tag{C.66}$$

where

$$L_t = \sum_{j=0}^{N-1}(1-\mathcal{F}_t^j)l_t^j \tag{C.67}$$

and σ_m is the productivity of the matching technology.

In this environment, there is a distinction between effective hours and measured hours. Effective hours is the hours of each person, adjusted by their productivity, a. Recall that the average productivity of a worker in working in cohort j (i.e., who has survived the endogenous productivity cut) is $\mathcal{E}_t^j/(1-\mathcal{F}_t^j)$. The number of workers who survive productivity cut in cohort j is $(1-\mathcal{F}_t^j)l_t^j$, so that our measure of total effective hours is

$$H_t = \sum_{j=0}^{N-1}\varsigma_{j,t}\mathcal{E}_t^j l_t^j. \tag{C.68}$$

In contrast, total measured hours is

$$H_t^{meas} = \sum_{j=0}^{N-1}\varsigma_{j,t}(1-\mathcal{F}_t^j)l_t^j.$$

The job finding rate is

$$f_t = \frac{m_t}{1-L_t}. \tag{C.69}$$

The probability of filling a vacancy is

[29] Note that I use the Cobb-Douglas specification of the matching function. This is not the case in den Haan, Ramey and Watson (2000, henceforth dHRW) where they use $m_t = \frac{(1-L_t)v_t}{((1-L_t)^l+v_t^l)^{\frac{1}{l}}}$ (with a parameter $l = 1.27$ calibrated for the US data). See Section 3.1 in the main text for comparison.

$$Q_t = \frac{m_t}{v_t}. \tag{C.70}$$

Total vacancies v_t are related to vacancies posted by the individual cohorts as follows:

$$v_t = \frac{1}{Q_t^{\iota}} \sum_{j=0}^{N-1} \tilde{v}_t^j (1 - \mathcal{F}_t^j) l_t^j.$$

Note however that this equation does not add a constraint to the model equilibrium because it can be derived from the equilibrium equations (C.70), (C.20) and (C.12).

C.3.7 Characterization of the bargaining set

Implicitly, it was assumed that the scaled wage,

$$w_t^i = \frac{W_t^i}{z_t^+ P_t},$$

paid by an employment agency which has renegotiated most recently i periods in the past is always inside the bargaining set, $[\underline{w}_t^i, \bar{w}_t^i]$, $i = 0, 1, \ldots, N-1$. Here, $\bar{w}_t^i$ has the property that if $w_t^i > \bar{w}_t^i$ then the agency prefers not to employ the worker and $\underline{w}_t^i$ has the property that if $w_t^i < \underline{w}_t^i$ then the worker prefers to be unemployed. We now describe our strategy for computing $\underline{w}_t^i$ and $\bar{w}_t^i$.

The lower bound, $\underline{w}_t^i$, sets the surplus of a worker, $(1 - \mathcal{F}_t^i)\left(V_{z^+,t}^i - U_{z^+,t}\right)$, in an agency in cohort i to zero. By (C.60),

$$\begin{aligned} U_{z^+,t} &= \underline{w}_t^i \varsigma_{i,t} \frac{1-\tau^y}{1+\tau^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L)\psi_{z^+,t}} \\ &\quad + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[\rho(1 - \mathcal{F}_{t+1}^{i+1}) V_{z^+,t+1}^{i+1} + (1 - \rho + \rho \mathcal{F}_{t+1}^{i+1}) U_{z^+,t+1}\right] \end{aligned}$$

for $i = 0, \ldots, N-1$. In steady state, this is

$$\underline{w}_t^i = \frac{U_{z^+} + \zeta^h A_L \frac{\varsigma_i^{1+\sigma_L}}{(1+\sigma_L)\psi_{z^+}} - \beta[\rho(1-\mathcal{F}^{i+1})V_{z^+}^{i+1} + (1-\rho+\rho\mathcal{F}^{i+1})U_{z^+}]}{\varsigma_i \frac{1-\tau^y}{1+\tau^w}}$$

where a variable without time subscript denotes its steady state value.

We now consider the upper bound, $\bar{w}_t^i$, which sets the surplus $J_{z^+,t}$ of an agency in cohort i to zero, $i = 0, \ldots, N-1$. From (C.57),

$$\begin{aligned} 0 &= \sum_{j=0}^{N-1-i} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} \left[\left(\bar{w}_{t+j} \frac{\mathcal{E}_{t+j}^j}{1 - \mathcal{F}_{t+j}^j} - G_{t,j} \bar{w}_t^i\right) \varsigma_{j,t+j} - \frac{\kappa}{\varphi} (\tilde{v}_{t+j}^j)^{\varphi}\right] \Omega_{t+j}^j \\ &\quad + \beta^{N-i} \frac{\psi_{z^+,t+N-i}}{\psi_{z^+,t}} J_{z^+,t+N-i} \frac{\Omega_{t+N-i}^{N-i}}{1 - \mathcal{F}_{t+N-i}^0} \end{aligned}$$

for $i = 0, \ldots, N-1$. In steady state,

$$0 = \sum_{j=0}^{N-1-i} \beta^j \left[\left(\bar{w} \frac{\mathcal{E}^j}{1-\mathcal{F}^j} - G_j \bar{w}^i \right) \varsigma_j - \frac{\kappa}{\varphi} (\tilde{v}^j)^{\varphi} \right] \Omega^j$$
$$+ \beta^{N-i} J_{z^+} \frac{\Omega^{N-i}}{1-\mathcal{F}^0}.$$

For the dynamic economy, the additional unknowns are the $2N$ variables composed of $\underline{w}_t^i$ and $\bar{w}_t^i$ for $i = 0, 1, \ldots, N-1$. We have an equal number of equations to solve them.

C.3.8 Summary of equilibrium conditions for the full model

This subsection summarizes the equations of the labor market that define the equilibrium and how they are integrated with the financial frictions model. The equations include the N efficiency conditions that determine hours worked, (C.31); the law of motion of the workforce in each cohort, (C.13); the FOCs associated with the vacancy decision, (C.35), (C.38), $j = 0, \ldots, N-1$; the derivative of the employment agency surplus with respect to the wage rate, (C.59); scaled agency surplus, (C.57); the value function of a worker, $V_{z^+,t}^i$, (C.60); the derivative of the worker value function with respect to the wage rate, (C.62); the growth adjustment term, $G_{t,j}$ (C.37); the scaled value function for unemployed workers, (C.56); the (suitably modified) resource constraint, (C.65); the equations that characterize the productivity cutoff for job separations, (C.50) and (C.54); the separations that characterize $\tilde{J}_{z^+,t}^j$ (C.55); the value of finding a job, (C.19); the job finding rate, (C.69); the probability of filling a vacancy, (C.70); the matching function, (C.20); the wage updating equation for cohorts that do not optimize, (C.32); the equation determining total employment, (C.67); the equation determining Ω_{t+j}^j, (C.58); the equation determining the hiring rate, χ_t^i (C.12); the equation determining the number of matches (the matching function), (C.66); the definition of total effective hours (C.68); the equations defining $\mathcal{M}_t^j$, (C.61); the equations defining $\mathcal{F}_t^j$, (C.30); the equations defining $\mathcal{E}_t^i$, (C.29); the equations defining $\mathcal{G}_t^{j\prime}$ (C.48); the equations defining $\mathcal{F}_t^{j\prime}$ (C.46).

The following additional endogenous variables are added to the list of endogenous variables in the financial frictions model:

l_t^j, $\mathcal{E}_t^j$, $\mathcal{F}_t^j$, $\varsigma_{j,t}$, $\mathcal{M}_t^j$, $\bar{a}_t^j$, $\tilde{v}_t^j$, $G_{t,j}$, Q_t, Ω_{t+j}^j, $J_{w,t}$, w_t, $J_{z^+,t}$, $V_{z^+,t}^j$, $U_{z^+,t}$, $V_{w,t}^0$, $V_{z^+,t}^x$, f_t, m_t, v_t, χ_t^j, $\tilde{\pi}_{w,t}$, L_t, $\mathcal{G}_t^{j\prime}$, $\mathcal{F}_t^{j\prime}$ and $\tilde{J}_{z^+,t}^j$

We drop the equations from the financial frictions model that determines wages, eq. (C.2), (C.71), (C.72), (C.73), and (C.74),

$$\mathring{w}_t = \left[(1-\xi_w)(w_t)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mathring{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}$$

$$= \left[(1-\xi_w) \left(\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right)^{\lambda_w} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mathring{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}} \quad \text{(C.71)}$$

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} \mathring{w}_t^{\frac{-\lambda_w}{1-\lambda_w}} h_t \frac{1-\tau^y}{1+\tau^w} + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \tag{C.72}$$

$$K_{w,t} = \zeta_t^h \left(\mathring{w}_t^{\frac{-\lambda_w}{1-\lambda_w}} h_t\right)^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \tag{C.73}$$

$$\frac{1}{A_L} \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}. \tag{C.74}$$

The equations which describe the dynamic behavior of the model are those of the financial frictions model plus those discussed in the employment frictions block. Finally, the resource constraint needs to be adjusted to include monitoring as well as recruitment (hiring) costs. Similarly, measured GDP is adjusted to exclude both monitoring costs and recruitment costs.

C.4 Measurement equations

Below we report the measurement equations we use to link the model to the data. Our data series for inflation and interest rates are annualized in percentage terms, so we make the same transformation for the model variables, i.e. multiplying by 400

$$\begin{aligned} R_t^{data} &= 400(R_t - 1) - \vartheta_1 400(R-1) \\ R_t^{*,data} &= 400(R_t^* - 1) - \vartheta_1 400(R^* - 1) \end{aligned}$$

$$\begin{aligned} \pi_t^{d,data} &= 400\log\pi_t - \vartheta_1 400\log\pi + \varepsilon_{\pi,t}^{me} \\ \pi_t^{c,data} &= 400\log\pi_t^c - \vartheta_1 400\log\pi^c + \varepsilon_{\pi^c,t}^{me} \\ \pi_t^{i,data} &= 400\log\pi_t^i - \vartheta_1 400\log\pi^i + \varepsilon_{\pi^i,t}^{me} \\ \pi_t^{*,data} &= 400\log\pi_t^* - \vartheta_1 400\log\pi^*, \end{aligned}$$

where $\varepsilon_{i,t}^{me}$ denote the measurement errors for the respective variables. In addition, $\vartheta_1 \in \{0,1\}$ allows us to handle demeaned and non-demeaned data. In particular, the data for interest rates and foreign inflation are not demeaned. The domestic inflation rates are demeaned.

We use undemeaned first differences in total hours worked,

$$\Delta\log H_t^{data} = 100\Delta\log H_t + \varepsilon_{H,t}^{me}.$$

We use demeaned first-differenced data for the remaining variables. This implies setting $\vartheta_2 = 1$ below:

$$\begin{aligned}
\Delta \log Y_t^{data} &= 100\left(\log \mu_{z^+,t} + \Delta \log \left[y_t - p_t^i a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} - d_t - \frac{\kappa}{2}\sum_{j=0}^{N-1}(\tilde{v}_t^j)^2(1-\mathcal{F}_t^j)l_t^j\right]\right) \\
&\quad - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{y,t}^{me} \\
\Delta \log Y_t^{*,data} &= 100(\log \mu_{z^+,t} + \Delta \log y_t^*) - \vartheta_2 100(\log \mu_{z^+}) \\
\Delta \log C_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log c_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{c,t}^{me} \\
\Delta \log X_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log x_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{x,t}^{me} \\
\Delta \log q_t^{data} &= 100\Delta \log q_t + \varepsilon_{q,t}^{me} \\
\Delta \log M_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log Imports_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
&= 100\left[\log \mu_{z^+,t} + \Delta \log \begin{pmatrix} c_t^m(\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} \\ +i_t^m(\mathring{p}_t^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} \\ +x_t^m(\mathring{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \end{pmatrix}\right] - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
\Delta \log I_t^{data} &= 100[\log \mu_{z^+,t} + \log \mu_{\psi,t} + \Delta \log i_t] - \vartheta_2 100(\log \mu_{z^+} + \log \mu_\psi) + \varepsilon_{I,t}^{me} \\
\Delta \log G_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log g_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{g,t}^{me}
\end{aligned}$$

note that neither measured GDP nor measured investment include investment goods used for capital maintenance. To calculate measured GDP we also exclude monitoring costs and recruitment costs.

The real wage is measured by the employment-weighted average Nash bargaining wage in the model:

$$w_t^{avg} = \frac{1}{L}\sum_{j=0}^{N-1} l_t^j G_{t-j,j} w_{t-j}\bar{w}_{t-j}.$$

Given this definition, the measurement equation for demeaned first-differenced wage is

$$\Delta \log(W_t/P_t)^{data} = 100\Delta \log \frac{\tilde{W}_t}{z_t^+ P_t} = 100(\log \mu_{z^+,t} + \Delta \log w_t^{avg}) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{W/P,t}^{me}.$$

Finally, we measure demeaned first-differenced net worth, interest rate spread and unemployment as follows:

$$\begin{aligned}
\Delta \log N_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log n_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{N,t}^{me} \\
\Delta \log Spread_t^{data} &= 100\Delta \log(z_{t+1} - R_t) = 100\Delta \log\left(\frac{\bar{\omega}_{t+1}R_{t+1}^k}{1-\frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}} - R_t\right) + \varepsilon_{Spread,t}^{me} \\
\Delta \log Unemp_t^{data} &= 100\Delta \log(1-L_t) + \varepsilon_{Unemp,t}^{me}
\end{aligned}$$

In a model with observed vacancies, vacancies are measured as the first difference of logged total vacancies.

References

[1] Christiano, Lawrence J., Mathias Trabandt and Karl Walentin, 2010. "Involuntary unemployment and the business cycle", NBER Working Papers 15801, National Bureau of Economic Research, Inc.